Flavor changing neutral current constraints on standard-like orbifold models

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Abstract

We examine for standard-like orbifold compactification models the constraints due to quarks and leptons generation non-universality of soft supersymmetry breaking interactions. We follow the approach initiated by Ibáñez and Lüst and developed by Brignole, Ibáñez and Muñoz . This is based on a locally supersymmetric $\sigma-$model action of moduli and matter fields obeying the stringy duality symmetries. It is assumed that the low energy fields of the minimal supersymmetric standard model are in one-to-one correspondence with string massless modes and that supersymmetry breaking takes place simultaneously with the lifting of flat directions for dilaton and compactification moduli fields. The breaking of supersymmetry is represented in terms of dilaton and moduli auxiliary field components and, consistently with a vanishing cosmological constant, is parametrized in terms of the dilaton-moduli mixing angle θ and the gravitino mass scale m_g. The soft supersymmetry breaking interactions (gaugino masses, squarks and sleptons mass matrices, scalars interactions A and B coupling constants) are calculable as a function of these parameters and of the discrete set of modular weight parameters specifying the modular transformation properties of the low energy fields. To establish the flavor changing neutral current constraints we solve the renormalization group one-loop equations for the full set of gauge, Yukawa and supersymmetry breaking coupling constants. A simplified version is used in which one treats the contributions from the third generation quarks and lepton Yukawa couplings exactly, while retaining for the first and second generations couplings only the leading order term in the large logarithm variable. Numerical results are obtained for the quantities $\Delta_{MN} = V_M \tilde{m}^2_{MN} V^\dagger_N$, corresponding to the mass matrices of squarks and sleptons in the super-CKM basis, for which experimental bounds can be determined via the super-box and super-penguin diagrams with gluino or neutralino exchange.

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1. Introduction

The string-based 4-d supergravity σ-models inherit from the duality symmetries of compactified strings (see ref.[1] for a review) an additional unbroken symmetry corresponding to discrete modular groups which act as Kähler transformations on the moduli and matter superfields [2]. The simplest case where a general analysis of the moduli space structure and of the modular symmetry action can be made is that of compactifications described by world sheet σ-models with (2,2) supersymmetry [3]. No results of comparable rigor exist for the phenomenologically more interesting (0,2) models, with or without Wilson lines. However, extensive analyses of these models exist within the orbifold toroidal compactification framework [4-8]. Here, string tree-level correlators are used to identify the Kähler potentials of the string massless moduli and matter modes and to determine modular transformation laws of the associated fields leading to assignment of modular weights [8]. String one-loop correlators [9,10] are also required to establish the cancellation of $\sigma-$model duality anomalies which takes place as a result of string mass threshold effects on gauge coupling constants in combination with an analog of the Green-Schwarz mechanism [11,12].

The restrictions placed by duality on supergravity σ-models greatly enhance the predictive power of studies of supersymmetry breaking within the so-called standard-like models, which are based on the assumption of a direct compactification to the minimal supersymmetric model. One important aspect is to narrow the choice of the functional dependence on the compactification moduli of the non-perturbative component of the superpotential which is supposed to lift flat directions in dilaton and moduli field space. The main hope, however, is that these non-vanishing VEVs (vacuum expectation values) of scalar fields components of dilaton and moduli are accompanied by non-vanishing VEVs for the corresponding auxiliary fields components, thus associating supersymmetry breaking with the same (as yet unknown) mechanisms that fix the gauge and Yukawa coupling constants of the low energy theory [13-15]. The viability of this picture has been examined in several recent works within the hidden sector gaugino condensation approach with a view to make contact with phenomenology [16,17]. However, to avoid occurrence of a vanishing dilaton auxiliary field component, $F_S = 0$, or a non-vanishing cosmological constant, elaborate versions must be used, in which hidden matter condensation or gaugino condensation from several gauge group factors play a role in the local supersymmetry breaking [18]. Motivated by these limitations, Brignole, Ibáñez and Muñoz [19] have proposed an alternative model independent approach avoiding any specific assumption on the non-perturbative superpotential except for the condition of non-vanishing auxiliary fields components of dilaton, moduli and possibly matter superfields, which are parametrized in such a way as to ensure a vanishing cosmological constant. While still rooted within the gaugino condensation framework, this approach is characterized by a stronger emphasis on auxiliary fields over scalar fields, thus providing a transparent representation of supersymmetry breaking in terms of mixing angles within the goldstino and the gravitino mass. However, the scalar VEVs stand out as extra free parameters in this approach.

The low energy limit of standard-like models can then be derived on minimal grounds by assuming a one-to-one correspondence between string massless modes and the low energy (quarks, leptons, Higgs) particles and, in the familiar way, expanding the supergravity $\sigma-$model model action in powers of the inverse Planck mass $1/M_P$. The supersymmetry breaking parameters (gaugino masses, scalars masses, bilinear and trilinear scalars interactions couplings) can be explicitly calculated in terms of the dilaton and moduli scalar and auxiliary components VEVs and of the fields modular weights.

One characteristic feature of the predicted soft supersymmetry breaking coupling constants is the suggested presence of large non-universal contributions with respect to the quarks and leptons generation quantum numbers [8]. Such generation dependence of squarks and sleptons mass matrices and of A-matrices for couplings of squarks and slep-

tons to Higgs bosons could contribute to flavor changing neutral current processes at an observable level, a fact which can be used under appropriate assumptions to set upper bouds on generation dependence [20,21]. However, the relevant physical observables in supergravity models are those obtained after summing the large logarithms contributed by radiative corrections. This requires solving the renormalization group flow equations down to the electroweak scale, using the soft parameters as boundary conditions at the string unification scale [22,23]. While the generation independent interactions of gauginos reduce whichever large non-universality present at the string scale, Yukawa interactions have an opposite effect. It is then mainly a quantitative issue to determine what residual non-universal dependence is not wiped out at the electroweak scale. Also, the fact that the quantities which enter in the evaluation of super-box or super-penguin diagrams involve the tranformation matrices which diagonalize the quarks and leptons mass matrices, implies that the relevant observables are also sensitive to the flavor structure of Yukawa interactions.

The issue of universality of supersymmetry breaking is closely tied with the flavor problem. Conceptually, it is difficult to reconcile the important flavor asymmetries present in Yukawa interactions of quarks and leptons with the postulate of a spontaneous supersymmetry breaking independent of flavor or generation quantum numbers [24]. Non-universality also poses an acute naturalness problem to supergravity models [25,26]. This arises in grand unified theories embedding of the minimal supersymmetric standard model where radiative corrections from decoupling of superheavy modes can induce contributions to mass matrices of scalar superpartners in generation space which are misaligned with those arising from Yukawa couplings of low energy modes [20]. The early envisaged possibility to limit the size of non-universal terms on the basis of flavor (horizontal) discrete or continuous symmetries has been revived recently in a number of interesting works [27-30]. Relaxing the universality restriction also plays an important role in widening the allowed parameter space of the minimal supersymmetric standard model [31]. Note however that string theory interpretation of non-universality differs from that of field theory in two respects: (i) Non-universality is associated with generation dependent assignment of modular weights to low energy fields taking discrete rational values rather than with the spontaneous breakdown of an horizontal symmetry; (ii) It sets in at the larger string unification scale which should ease its smearing by flavor-blind radiative corrections. Thus, string theory provides us with a framework to accommodate large non-universality as well as mechanisms to reduce their observable effects. Related recent works on this subject are refs.[32,33]

Our main goal in this work is to examine the implications of the experimental flavor changing neutral current bounds [20,34] concerning the presence of a generation non-universality in the dimension-two scalars mass matrices, $\tilde{m}^2_{MN}, [M, N = L, R]$, and the dimension-three scalars couplings matrices $A^x_{ij}, [x = u, d, e]$. In the super-CKM basis, with flavor diagonal Yukawa couplings of quarks and squarks or leptons and sleptons to gauginos, the generation dependence in the super-box or super-penguin diagrams, propagating superpartners of low energy particles, involve the quantities, $\Delta_{MN} = V_M \tilde{m}^2_{MN} V^\dagger_N$, where $V_M, [M = L, R]$ are transformations which diagonalize the quarks and leptons mass matrices. We shall restrict consideration to the standard-like orbifold compactification leading to the minimal supersymmetric standard model . The local supersymmetry breaking parameters are then determined as a function of the dilaton and moduli auxiliary and scalar fields components at the large string unification scale. To compare with upper bounds on the parameters Δ_{MN}, we need to integrate the scale evolution equations. As is well known, the corrections from gauge bosons and fermions loops, being flavor-blind, cause important dilution of flavor mixing effects [26]. Thus, non-universality should be strongly suppressed in a dilatonic goldstino vacuum, $\cos\theta \approx 0$ [19]. A naive estimate [19] including only gauge interactions in the scale evolution yields a large upper bound on the cosine of the mixing

angle, $|\cos\theta| \leq 1/\sqrt{3}$, corresponding to a narrow range of values for the dilaton-moduli mixing angle, $90^\circ \leq \theta \leq 55^\circ$. An independent source of flavor mixing is also present in the trilinear scalar interactions, which are affected too by the renormalization group evolution.

The contents of the paper are organized in 4 sections. In Section 2, we recall the main ingredients of supergravity $\sigma-$model effective actions incorporating duality symmetries, as developed by Ibáñez and Lüst [8] (Subsection 2.1), and of supersymmetry breaking in this framework, using the parametrization of Brignole, Ibáñez and Muñoz [19] (Subsection 2.2). Next, we present the approximate version used for the one loop renormalization group equations in which the contributions from the third generation particles (top and bottom quarks and τ-lepton) are included exactly, while those of the first and second generations are included perturbatively (Subsection 2.3). Finally we discuss the choice at string scale for the Yukawa coupling matrices (Subsection 2.4). In Section 3, we present and discuss our numerical results. In Section 4, we summarize our main conclusions and suggestions for a future improved treatment of the problem.

2. Duality symmetry and supersymmetry breaking interactions

2.1 $\sigma-$model effective action

By moduli space(s) of a superstring theory one means the configuration space sector(s) of the associated world sheet field theory spanned by marginal operators deformations preserving the superconformal symmetry. This space constitutes the vacuum manifold of the low energy 4-d effective field theory. The moduli space of string-based σ-models supergravity are parametrized by: (1) the space-time dilaton field, $S(x)$; (2) the compactification moduli fields, $M(x), \bar{M}(x)$, associated to deformations of the Kähler and complex structures of the 6-d internal space manifold; (3) the matter-like background fields, A^W_γ, associated to Wilson gauge flux lines winding around non-contractible loops in the lattices of $E_8 \times E_8$ root space and internal 6-d space-time and other possible massless fields associated with duality [6]. Orbifolds (see refs.[35] for reviews) $\Omega = R^6/(\Lambda \times P)$ represent special exact solutions for an internal space given by a 6-torus of lattice Λ with a discrete point symmetry group P generated by elements of the $SU(3)$ holonomy subgroup of the 6-d tangent space group $SO(6)$. The point groups of abelian, Z_N or $Z_N \times Z_M$, orbifolds are generated by elements which can be represented in a complex basis as: $\theta = diag(e^{2\pi i\theta_i})$, $[\sum_i \theta_i = 1]$ with complex conjugate indices $i, \bar{i} = 1, 2, 3$ labeling coordinates, $X^{i=1,\bar{1}} = \frac{1}{\sqrt{2}} X^{\mu=1\pm i2}, \cdots$, and spinors. The relevant compactification moduli fields constitute then a subset of the moduli matrices, $[T_{i\bar{j}}, \bar{T}_{i\bar{j}}]$, $[U_{i\bar{j}}, \bar{U}_{i\bar{j}}]$, $[i, j = 1, 2, 3]$, invariant under P. For convenience, we shall restrict ourselves to the so-called generic orbifold case, characterized as allowing only diagonal Kähler structure moduli, $T_i, \bar{T}_i[i = 1, 2, 3]$, or a subset of these, while excluding entirely complex structure moduli $U_i, \bar{U}_i$ and matter background fields A^W_γ. (See ref.[9] for the list of allowed moduli and ref.[8] for the list of $\sigma-$model moduli spaces). As in most of the recent studies, we also shall specialize to decomposable 6-d tori with a metric tensor given by a direct product of metric tensors for three orthogonal 2-d tori, or complex planes, labelled by indices $i, \bar{i} = 1, 2, 3$. (Calculations with non-decomposable tori are considered in refs.[36,37]).

The Kähler potential of the effective $N = 1$ supergravity effective action can be derived in general form for the case of world sheet with (2,2) supersymmetry [3]. The part involving untwisted moduli is given in exact form as:

$$K_{moduli}(S, \bar{S}, M, \bar{M}, A^W_\gamma, A^{W\dagger}_\gamma) = -\sum_i \log\left(T_i + \bar{T}_i - \sum_\gamma A^{W\dagger}_\gamma \prod_a e^{2g_a V_a(R_\gamma)} A^W_\gamma\right)$$

$$-\sum_i \log(U_i+\bar{U}_i)-\log(S+\bar{S}), \tag{1}$$

while the part involving untwisted and twisted sectors matter superfields, denoted generically by $A_\alpha = A_\alpha + \theta\chi_\alpha + \theta^2 F_\alpha$, where the index α designates whatever quantum numbers (gauge, family,...) are needed to characterize string modes, is given in an expansion in powers of $\frac{A_\alpha}{M_P}$ as:

$$K_{matter}(M,\bar{M},A_\alpha,A_\alpha^\dagger)=\sum_\alpha\prod_i(T_i+\bar{T}_i)^{n_\alpha^i}\prod_m(U_m+\bar{U}_m)^{l_\alpha^m}\left(A_\alpha^\dagger\prod_a e^{2g_aV_a(R_\alpha)}A_\alpha\right)+\cdots, \tag{2}$$

where n_α^i, l_α^m are fields modular weights, rational numbers characterizing the modular group transformation laws. For each matter field, we have inserted its minimal gauge coupling interaction, such that each group factor G_a, with coupling constant g_a, is associated the real superfield $V_a(R_\alpha)=-iV_a^{a'}T^{a'}(R_\alpha)$, with $T^a(R_\alpha)$, the Lie algebra generators in the representation R_α subject to the normalization convention $Tr(T_aT_{a'})=\frac{1}{2}c(R_\alpha)\delta_{aa'}$, where $c(R_\alpha)$ is the Dynkin index of representation R_α. Note that the absence of off-diagonal kinetic terms in eq.(2) is an exact result for (2,2) models which is expected to remain true in general cases. The above formulas are found to hold for the wider class of world sheet (0,2) models only for orbifolds. We shall restrict ourselves to this case in the sequel. The duality symmetries are realized for the diagonal moduli, $T_{i\bar{i}}=T_i$, by the modular symmetry group $\Gamma=\left[SL(2,Z)\right]^3$ (or an appropriate subgroup), whose action on the moduli and matter fields in the supergravity basis (as opposed to the string vertex operators basis) [6] is specified by the transformations:

$$S\to S,\qquad T_i\to\frac{a_iT_i-ib_i}{ic_iT_i+d_i},\quad A_\alpha\to\prod_i(ic_iT_i+d_i)^{n_\alpha^i}A_\alpha, \tag{3}$$

where $(a_i,b_i,c_i,d_i)\in Z,\quad a_id_i-b_ic_i=1$, for $i=1,2,3$, define elements of $SL(2,Z)_{T_i}$ and the modular weights are given as [8]: $n_\alpha^i=-\delta_{\alpha i}$, (untwisted sector $\alpha=i=[1,2,3]$) and $n_\alpha^i=-(1-\theta^i+p^i-q^i)$, (twisted sectors), p^i,q^i being the number of complex conjugate left-moving sector oscillator excitations of type $\alpha^i_{m-\theta^i},\tilde{\alpha}^i_{m+\theta^i}$. Note that the background fields A_γ^W transform as untwisted states and that for complex structure moduli analogous formulas hold for the associated modular weights l_α^m with the interchange of p^i with q^i.

Each gauge symmetry group factor G_a of the full gauge group, $\prod_a G_a$, is associated a holomorphic gauge function $f_{aa'}$, defined in the adjoint representation. In the globally supersymmetry limit, with the field strength chiral superfield defined as, $-iW_\alpha^aT^a=\frac{1}{4}\bar{D}^2e^{-V_a}D_\alpha e^{V_a}=-\frac{i}{4}(i\lambda_\alpha^a+\cdots)T^a$, this contributes the familiar Lagrangian:

$$L_{gauge}=\frac{1}{4}\sum_a\int d^2\theta W^{a\alpha}W_\alpha^{a'}f_{aa'}(S,T_i)+c.c., \tag{4}$$

such that $f_{aa'}(S,M_i)=k_aS\delta_{aa'}$, with the dilaton VEV being related to the string coupling constant as $<S>=1/g_X^2$ and k_a being the level of the world sheet Kac-Moody algebra for the group G_a. The only requirement from duality on the superpotential is that it be a modular form of $SL(2,Z)_{T_i}$ of weight -1, corresponding to the modular group acting as the Kähler transformation, $K\to K+f_i+\bar{f}_i, W\to e^{-f_i}W,\quad [f_i=\log(ic_iT_i+d_i),\quad \bar{f}_i=f_i^\star]$.

The consideration of string and $\sigma-$model loop expansions provides important information on the moduli dependence of the effective action. Realization of duality as a Kähler transformation, entails the presence of chiral $U(1)_{T_i}$ R-symmetries transforming matter and gauge fermions as, $\psi \to \psi e^{-iw_i(\psi) Im f_i}$, with weights $w_i(\lambda^a) = \frac{1}{2}, \quad w_i(\chi_\alpha) = -\frac{1}{2}(1+2n^i_\alpha)$. For the diagonal modular subgroup, $f = \sum_i f_i, w(\lambda_a) = \frac{1}{2}, w(\chi_\alpha) = -\frac{1}{2}(1+\frac{2}{3}n_\alpha), \quad n_a = \sum_i n^i_\alpha$. At the quantum level these symmetries are affected by $\sigma-$model triangle anomalies involving non-local couplings of gauge bosons to Kähler and matter composite connections,

$$L_{nl} = -\frac{1}{4}\sum_a \int d^2\theta W^a W^a \frac{1}{(4\pi)^2}\frac{\bar{D}^2 D^2}{16\Box}\left[\frac{1}{2}\Big(c(G_a) - \sum_{R_\alpha} c(R_\alpha)\Big) K_{mod} + \sum_{R_\alpha} c(R_\alpha) \log det_{\alpha\bar{\beta}} (K_{mat})_{\alpha\bar{\beta}}\right] + c.c.$$

(Analogous $\sigma-$model gravitational anomalies are also present.) The duality anomalies contain a gauge group independent (universal) part which can be cancelled by an analog of the Green-Schwarz mechanism [11,12]. This is implemented in the dilaton chiral multiplet formulation by changing the dilaton field variable to [8,12]: $S \to S^{new} = S - \frac{1}{(4\pi)^2}\sum_i \delta^i_{GS} \log(T_i + \bar{T}_i)$, such that the new field transforms under modular transformations as: $S^{new} \to S^{new} + \frac{2}{(4\pi)^2}\sum_i \delta^i_{GS}(f_i - \bar{f}_i)$, while S is inert. Performing the implied substitution in eq(1), and suppressing the subscript 'new', the corrected dilaton Kähler potential becomes:

$$\log(S+\bar{S}) \to \log(Y) \equiv \log\left(S + \bar{S} + \frac{2}{(4\pi)^2}\sum_i \delta^i_{GS}\log(T_i + \bar{T}_i)\right), \tag{5}$$

where the modular invariant function Y is interpreted as an effective dilaton field whose VEV defines a renormalized string coupling constant field, $< Y >= 2/g^2_X$. At string one-loop level, there also arises mass threshold corrections in the gauge coupling constants which, after changing the dilaton field variable in the tree level term, leads to corrected gauge functions [9]:

$$f_{aa'}(S, T_i) \equiv f_a(S, T_i)\delta_{aa'} = \delta_{aa'}\left[k_a S + \sum_i \frac{\tilde{b}'^i_a}{(4\pi)^2}\log \eta^4(T_i)\right], \tag{6}$$

where $\eta(T)$ is Dedekind function (automorphic modular form of weight $\frac{1}{2}$) and $\tilde{b}'^i_a = b'^i_a - k_a\delta^i_{GS}$, with $b'^i_a = \frac{1}{2}[c(G_a) - \sum_\alpha (1+2n^i_\alpha)c(R_\alpha)]$, corresponding to the β-function slope parameters for the N=2 suborbifolds associated with the subgroup of the point symmetry group leaving the $i-$plane fixed. Using eqs.(1) and (2) to write the above non-local effective Lagrangian for the $\sigma-$model modular anomalies as:

$$L_{nl} = -\frac{1}{4(4\pi)^2}\sum_a \int d^2\theta W^a W^a \frac{\bar{D}^2 D^2}{16\Box}\sum_i b'^i_a \log(T_i + \bar{T}_i) + c.c.,$$

we see that the modular transform of L_{nl} is composed of a moduli-dependent term which is cancelled by threshold corrections (second term in the gauge function, eq.(6)) and a

universal moduli-independent term which is cancelled by virtue of the Green-Schwarz dilaton-moduli mixing (first tree level term in eq.(6)). Parenthetically, we note that an analogous cancellation also takes place for the QCD triangle anomalies affecting the Kähler R-symmetries $U(1)_{T_i}$ which are compensated by the breaking term occurring in threshold corrections, $(\theta_a)_{thres} = -(4\pi)^2 Im f_a(S,T_i) = -b_a^{'i} \log\left(\frac{\eta(T_i)}{\bar{\eta}(T_i)}\right)^2$. This term arises [37] because any modular invariant coupling term quadratic in colored fields can be transfered via a chiral rotation into a contribution to the θ-vacuum parameter, whose variation under $U(1)_{T_i}$, $\delta_i\theta_a = \frac{1}{2}\left(c(G_a) - \sum_\alpha(1+2n_\alpha^i)c(R_\alpha)\right)(f_i - \bar{f}_i)$, is exactly canceled by that of threshold corrections, $\delta_i(\theta_a)_{thres} = -b_a^{'i}(f_i - \bar{f}_i)$.

To simplify the discussion of moduli dependence, it is convenient to limit the number of unknowns by restricting to a one-dimensional direction in moduli space, corresponding to the overall volume modulus field $T(x)$. For an isotropic orbifold with three equivalent 2-tori, the overall modulus is defined by the restriction: $T_1(x) = T_2(x) = T_3(x) = T(x)$, and transforms under the diagonal subgroup $SL(2,Z)$ of $\left[SL(2,Z)\right]^3$, whose modular weights are given by the sums, $n_\alpha = \sum_i n_a^i$. The overall modulus for an anisotropic orbifold can be defined analogously by the condition: $r_1T_1 = r_2T_2 = r_3T_3 = T$, where r_i are real parameters characterizing the geometrical shape of the 6d-torus which obey $\prod_i r_i = 1$, by virtue of volume conservation. The squeezed orbifold with $T_1 \gg T_2, T_3$ has $r_1 \ll r_2, r_3$. The diagonal moduli VEVs are related to the radii of the 2-d tori as: $Re(T_i) = c_N R_i^2/((4\pi)^2)\alpha')$, with α' the string slope parameter and $c_N^2 \propto det(G)$, with G the 6-torus metric tensor. The c_N are calculable constants depending on the Z_N orbifold order. For instance, $c_3 = c_6 = \sqrt{3}, \quad c_4 = 2\sqrt{2}$. (Applications dealing with non-decomposable 6-d tori are developed in refs.[36].) Naturally, the values at which these radii, or the corresponding overall modulus and anisotropies, settle is determined by compactification dynamics which lies outside the present framework. However, because compactification means a choice of vacuum, minimization of the supergravity scalar potential will place certain conditions on these VEVs. Thus, for decomposable tori, the equivalence of different planes imposes a discrete permutational symmetry among the allowed moduli. An anisotropic solution could then arise in either of the following two situations: (1) A subset of the diagonal moduli is allowed. The Z_4 orbifold, for example, allows for two diagonal moduli fields only; (2) Spontaneous breakdown of the permutational symmetry. Nevertheless, according to ref.[14], it is not easy to find generic moduli configurations which realize the latter possibility.

Let us briefly summarize the main conclusions of Ibáñez and Lüst [8] concerning the range of values that can be assigned to modular weights. Given an orbifold, with point group in the allowed list of abelian Z_N or $Z_N \times Z_M$ groups [38], this is associated discrete sets of twists $\theta_i^{(p)}, [p = 1, \cdots, T]$ oscillators modings, Kac-Moody levels, and fixed points $f_i^{(q)}, [q = 1, \cdots, F]$. While Yukawa couplings depend on all these items, modular weights are essentially fixed by the first three only. The larger are the values assigned to k_a and to oscillator excitations, the wider is the range of modular weights. The main issue for minimal supersymmetric standard model compactification is to assign modular weights to the low energy fields consistently with cancellation of duality anomalies and with gauge coupling constants unification. An important characteristic of orbifolds here is the number of overall unrotated planes (i.e., planes left fixed by sweeping through the entire set of twists). This number can take the values: $N_{unr} = 3, 2, 1, 0$. The orbifolds $Z_N \times Z_M$ have $N_{unr} = 3$; Z_6' has $N_{unr} = 2$; $Z_4, Z_6, Z_8, Z_8', Z_{12}, Z_{12}'$ all have $N_{unr} = 1$; prime number orbifolds, Z_3, Z_7, have $N_{unr} = 0$. The duality anomalies cancellation places

the constraint, $\tilde{b}_a^{'i} = 0$, for overall rotated planes, $i = 1, 2, 3$, where the index a labels the standard model group factors, $a = 3, 2, 1$, corresponding to $SU(3) \times SU(2) \times U(1)$. For the minimal supersymmetric standard model the conditions $\tilde{b}_a^{'i} = 0$ for an overall rotated plane, take the form: $b_a^{'i} \equiv -c_a - \sum_g c_a^\alpha n^i_{\alpha_g} = k_a \delta^i_{GS}$, where g is a generation index, $c_3 = 3, c_2 = 5, c_1 = 11$ and c_a^α are rational numbers, which for standard model fields labeled in the following order, $\alpha = [Q, U^c, D^c, L, E^c, H_1, H_2]$, are given by [8]: $c_3^\alpha = [2, 1, 1, 0, 0, 0, 0]$, $c_2^\alpha = [3, 0, 0, 1, 0, 1, 1]$, $c_1^\alpha = [\frac{1}{3}, \frac{8}{3}, \frac{2}{3}, 1, 2, 1, 1]$. These modular anomalies cancellation conditions are absent for the $Z_N \times Z_M$ orbifolds, because of $N_{unr} = 3$, and involve one equation of the above type (for each $a = 3, 2, 1$) for the Z_6' orbifold, two for all other non-prime Z_N orbifolds and three for prime orbifolds.

The δ^i_{GS} are extra parameters which are calculable on a model-by-model basis, for example, by identifying $\tilde{b}_a^{'i} = b_a^{'i} - k_a \delta^i_{GS}$ to the slope parameter of the $N = 2$ suborbifold generated by subgroups of the point symmetry group leaving the i-plane fixed. Since threshold corrections are non-vanishing for overall unrotated planes only, it follows that the unification constraints act in complementary manner to the anomalies cancellation constraints, yielding stronger conditions for orbifolds with larger N_{unr}. Of course, it is always possible to partially relax these constraints by considering anisotropic orbifolds, since a condition such as $T_1 \gg T_2, T_3$, which freezes T_2, T_3, renders ineffective any threshold corrections involving these moduli. The analysis of Ibáñez and Lüst [8] indicates that most Z_N orbifolds fail to satisfy the combined anomalies cancellation and gauge unification constraints, unless one considers higher Kac-Moody levels for the non-abelian groups, which is not a favored option in orbifolds constructions. The acceptable canditates, within the minimal framework, are: Z_8', $Z_2 \times Z_3$ and $Z_2 \times Z_6'$ orbifolds. Generalization to squeezed orbifolds extends the acceptable orbifolds to the whole $Z_N \times Z_M$ set. Recall here that anisotropic compactification could also represent a viable mechanism to generate large hierarchies in the Yukawa couplings matrices [39]. Unlike the superpotential and gauge functions, the Kähler potential is negligibly affected by anisotropies. Indeed, the constraint $\sum_i \log r_i = 0$ removes all dependence here on r_i at tree level. At one-loop level, a residual contribution may arise from the term, $\sum_i \delta^i_{GS} \log(T_i + \bar{T}_i) = -\sum_i \delta^i_{GS} \log r_i + \delta_{GS} \log(T + \bar{T})$, which can be absorbed inside the dilaton field. However, whereas threshold effects involve components δ^i_{GS} for overall unrotated planes, the one loop Kähler potential involves all planes, whether rotated or not.

2.2 Soft supersymmetry breaking terms

The scalar potential of the N=1 supergravity $\sigma-$model is composed of F- and D-terms, $V = V_F + V_D$ [40], defined as:

$$V_F = e^K \big(D_i W (K^{-1})^{i\bar{j}} D_{\bar{j}} W - 3|W|^2\big) = (F_i G^{i\bar{j}} F_{\bar{j}} - 3e^G), \quad V_D = \frac{1}{2} D_a (f^{-1})_{aa'} D_{a'}, \quad (7)$$

where $G = K + \log |W|^2$, $D_i W = W_i + K_i W \equiv \partial_i W + W \partial_i K$, $D_a = G_i (T^a)_{ij} A_j$, the moduli and matter fields are labeled by the index i and $F_i = e^{G/2} (G^{-1})_{i\bar{j}} G_{\bar{j}}$ identify with their auxiliary components. Following the hidden sector approach, we assume that the superpotential includes a non-perturbative part which lifts flat directions in S and T_i by inducing a non-trivial minimum of V characterized by non-vanishing VEVs for dilaton and moduli scalar S, T_i as well as auxiliary field components F_S, F_{T_i}. We shall follow the approach of Brignole, Ibáñez and Muñoz [19] in which local supersymmetry breaking with $< V >= 0$ is assumed to be saturated by the dilaton and moduli auxiliary fields. Generalizing the parametrization of ref.[19], we represent these quantities as follows:

$$(G^{\frac{1}{2}})_{S\bar{S}} F_{\bar{S}} = \sqrt{3} m_g e^{i\alpha_S} \sin\theta, \quad (G^{\frac{1}{2}})_{T_i \bar{T}_j} F_{\bar{T}_j} = \sqrt{3} m_g e^{i\alpha_T} e_i \cos\theta, \quad (8)$$

under the restriction $\sum_i e_i^2 = 1$, necessary to ensure $< V >= 0$, and the simplifying assumption that the anisotropy parameters $e_i [i = 1, 2, 3]$ are real. The gravitino mass is $m_g = e^{G/2}$ and θ is interpreted as a dilaton-moduli mixing angle. The complex phases α_S, α_T, along with possible non-vanishing imaginary parts of VEVs, ImS, ImT_i, represent CP violation parameters. The isotropic case, which is considered in ref.[19], can be recovered by identifying the three moduli components, hence setting $F_{T_i} = F_T, \quad e_i = 1/\sqrt{3}$.

The case involving the one-loop corrected Kähler potential, eq.(5), can be treated in close analogy with the above tree level case by modifying the parametrization as follows [19]:

$$\frac{1}{Y}\left(F_S + \frac{2}{(4\pi)^2}\sum_i \frac{\delta^i_{GS} F_{T_i}}{T_i + \bar{T}_i}\right) = \sqrt{3} m_g e^{i\alpha_S}\sin\theta, \quad \left(1 + \frac{2\delta^i_{GS}}{(4\pi)^2 Y}\right)^{\frac{1}{2}} \frac{F_{T_i}}{T_i + \bar{T}_i} = \sqrt{3} m_g e^{i\alpha_T} e_i \cos\theta. \tag{9}$$

The isotropic case results by setting, $e_i = \frac{1}{\sqrt{3}}, \delta^i_{GS} = \frac{\delta_{GS}}{3}, \delta^i_{GS}\frac{F_{T_i}}{T_i + \bar{T}_i} = \delta_{GS}\frac{F_T}{T + \bar{T}}$. The parametrization (8) can also be generalized to allow for a matter component of the Goldstino by introducing an additional mixing angle θ_A and writing [19]:

$$\begin{aligned} (G^{\frac{1}{2}}F)_S =& \sqrt{3} m_g e^{i\alpha_S}\sin\theta\cos\theta_A, \quad (G^{\frac{1}{2}}F)_{T_i} = \sqrt{3} e^{i\alpha_T} m_g e_i \cos\theta\cos\theta_A, \\ (G^{\frac{1}{2}}F)_A =& \sqrt{3} m_g e^{i\alpha_A}\sin\theta_A, \end{aligned} \tag{10}$$

the one loop case involving again a direct generalization of eq.(9). Auxiliary and scalar components are not independent, of course. They might be related by specifying the non-perturbative superpotential. Consider, for instance, the gaugino condensation approach, in which approximate elimination of the composite gauge fields associated to a subset $\prod_{A'} G'_A$ of hidden gauge group factors, leads to the following toy model superpotential [13-15],

$$W_{np}(S, T_i) = \sum_A h_A e^{-\frac{3k_A S}{2b_A}} \prod_i \left[\eta(T_i)\right]^{\nu_i}, \tag{11}$$

where h_A are constant parameters and $\nu_i = -2$ as required to obtain a modular invariant function, $G_{np} = K + \log|W_{np}|^2$, free of singularities in the T_i-planes. By virtue of modular invariance, the field space of the variables T can be restricted to the fundamental region, $F = [|T| > 1, Im(T) < \frac{1}{2}]$. The relation between anisotropy parameters e_i and the r_i parameters, defined by $r_i T_i(x) = T(x)$, can then be obtained by calculating the auxiliary fields associated with eq.(11):

$$(G^{\frac{1}{2}}F)_{T_i} = m_g\left[-1 + \nu_i(T_i + \bar{T}_i)\frac{d\log\eta(T_i)}{dT_i})\right] = m_g\left[-1 - \frac{\pi}{12}\nu_i(T_i + \bar{T}_i)\left(1 - 24\sum_{n=1}^{\infty}\frac{nq_i^n}{1 - q_i^n}\right)\right]. \tag{12}$$

Since for $T_i \in F, \quad Re(T_i) > \sqrt{3}/2$, and the moduli dependence involves the variable $q_i = e^{-2\pi T_i}$, we see from the size of the exponential suppression, $q_i < e^{-\pi\sqrt{3}} \approx 4.3310^{-3}$, that the relevant regime where the infinite sum in eq.(12) needs to be evaluated is that of large T_i. Assuming that all T_i lie in the large compactification radius regime, one finds: $\frac{e_i}{e_j} \approx \frac{r_j}{r_i}$. For an isotropic orbifold, the restriction of equal $r_i = r = 1$, likewise implies equal $e_i = 1/\sqrt{3}$. For an anisotropic orbifold, say, with $T_1 \gg T_2, T_3$, or $r_1 \ll r_2, r_3$, assuming the large-T limit, leads to: $e_1 \gg e_2, e_3$.

Let us now turn to the implications concerning the soft supersymmetry breaking interactions. We identify the string low energy observable sector with that of the minimal supersymmetric standard model and assume the presence of a perturbative superpotential describing the renormalizable trilinear and bilinear Yukawa couplings of quarks, leptons and Higgs bosons left chiral superfields,

$$W=\lambda^u_{ij}Q_i\epsilon H_2U^c_j+\lambda^d_{ij}Q_i(-\epsilon)H_1D^c_j+\lambda^e_{ij}L_i(-\epsilon)H_1E^c_j+\mu H_1\epsilon H_2, \tag{13}$$

such that $Q_i=\binom{u_i}{d_i}, L_i=\binom{\nu_i}{e_i}, H_1=\binom{H_1^0}{H_1^-}, H_2=\binom{H_2^+}{H_2^0}$ denote quarks, leptons and Higgs weak doublets left-chirality fields, with the invariant combination, $Q\epsilon H=Q_\alpha\epsilon_{\alpha\beta}H_\beta, \quad \epsilon_{\alpha\beta}\quad [\epsilon_{12}=1]$ being the antisymmetric symbol; U^c, D^c, E^c denote antiquarks and antilepton weak singlets left-chirality fields and the indices $i,j=[1,2,3]$ label the squarks or sleptons generations. We assume a one-to-one correspondence with massless string modes, so that modular symmetry implies that Yukawa coupling constants $\lambda^x_{ij}(T_k), [x=u,d,e]$ transform under $SL(2,Z)_m$ as modular forms of weight, $(-1-n^m_{Qi}-n^m_{Uj}-n^m_{H_2})$ for $x=u$ and similar formulas for other cases. For a given orbifold, these are calculable functions of the moduli fields depending on characteristics of states, such as oscillator excitations, reference plane and, for twisted sector states, coordinates of fixeds points [39,41]. Adding the perturbative superpotential (13) to the non-perturbative one and substituting in the scalar potential, eq.(7), yields a tree level soft supersymmetry breaking potential of the familiar form,

$$\begin{aligned}V_{soft}=\tilde m^2_\alpha A_\alpha A^\dagger_\alpha+\frac{1}{2}M_a\bar\lambda_a\lambda_a+m_g\Bigg[\Bigg(&A^u_{ij}\lambda^{ij}_uQ_i\epsilon H_2U^c_j+A^d_{ij}\lambda^{ij}_dQ_i(-\epsilon)H_1D^c_j\\&+A^e_{ij}\lambda^{ij}_eL_i(-\epsilon)H_1E^c_j+B_\mu\mu H_1\epsilon H_2\Bigg)+c.c.\Bigg],\end{aligned} \tag{14}$$

with the following formulas for the soft breaking coupling contants in the tree level case[19],

$$\begin{aligned}M_a=&\frac{m_g}{Re(f_a)}\left[\sqrt{3}e^{-i\alpha_S}\sin\theta(\frac{\partial f_a}{\partial S})Re(S)-\frac{1}{16\pi^3}e^{-i\alpha_T}\cos\theta\sum_i\tilde b'^i_aRe(T_i)\hat G_2(T_i)\right],\\ \tilde m^2_\alpha=&m^2_g(1+n_\alpha{}''\cos^2\theta),\\ A_{\alpha\beta\gamma}=&-\sqrt{3}e^{-i\alpha_S}\sin\theta-e^{-i\alpha_T}\cos\theta\Bigg(e'+n'_\alpha+n'_\beta+n'_\gamma-\sqrt{3}\sum_i(T_i+\bar T_i)e_i\frac{w^{T_i}_{\alpha\beta\gamma}}{w_{\alpha\beta\gamma}}\Bigg),\\ B_\mu=&-1-\sqrt{3}e^{-i\alpha_S}\sin\theta\Bigg(1-\frac{\mu_S}{\mu}(S+\bar S)\Bigg)\\ &-e^{-i\alpha_T}\cos\theta\Bigg(e'+n'_{H_1}+n'_{H_2}-\sqrt{3}\sum_ie_i(T_i+\bar T_i)\frac{\mu_{T_i}}{\mu}\Bigg).\end{aligned} \tag{15}$$

The index α attached to low energy matter fields, denoted generically as $A_\alpha(x)$, subsumes gauge and generation quantum numbers and the index $a=3,2,1$, attached to gaugino fields, $\lambda_a(x)$, labels the standard model gauge group factors $SU(3)\times SU(2)\times U(1)$, in the indicated order. We have introduced the following notations: $n^{''}_\alpha=3\sum_ie^2_in^i_\alpha, \quad n^{'}_\alpha=$

$\sqrt{3}\sum_i e_i n^i_\alpha$, $\quad e' = \sqrt{3}\sum_i e'_i, \hat{G}_2(T) \equiv G_2(T) - \frac{2\pi}{T+\bar{T}}$, $\quad G_2(T) = -4\pi\frac{\eta'(T)}{\eta(T)} \approx \pi^2\big(1/3 - 8\sum_n nq^n/(1-q^n)\big)$; $w_S = \partial w/\partial S$, with $w = w_{\alpha\beta\gamma}A_\alpha A_\beta A_\gamma$ a generic notation for the superpotential terms in eq.(13). The isotropic orbifold case, which is treated in ref.[19], is recovered by setting, $e_i = 1/\sqrt{3}$, which gives: $e' = 3, n''_\alpha = n'_\alpha = n_\alpha = \sum_i n^i_\alpha$, $\quad \sqrt{3}\sum_i e_i(T_i + \bar{T}_i)f_{T_i} = (T+\bar{T})f_T$. The one loop improved case is described by identical formulas to eqs.(15) with the substitution:

$$e_i \rightarrow \frac{e_i}{\left(1 + \frac{2\delta^i_{GS}}{(4\pi)^2 Y}\right)^{\frac{1}{2}}}.$$

The isotropic one loop improved case is recovered by performing the substitutions described above for the tree level case, together with the formal replacement[19]:

$$\cos\theta \rightarrow \cos\theta(1 + \frac{\delta_{GS}}{24\pi^2 Y})^{-\frac{1}{2}}, \quad \sin\theta \rightarrow \sin\theta.$$

2.3 Renormalization group analysis

Distinct physical theories are parametrized by renormalization group trajectories which trace the scale dependence of the gauge, Yukawa and soft supersymmetry breaking coupling constants . These are described by first-order differential equations in the scale variable, $t = \log\frac{M_X^2}{Q^2}$, with M_X the string unification mass scale and Q a floating renormalization scale, so that a solution to these equations is fixed uniquely once boundary conditions are specified at M_X, as for the quantities quoted in eqs.(15), or at some other scale. While results for the renormalization group equations are available in the literature up to two-loop order, in the present work, we shall restrict ourselves to the one-loop equations, and rely on refs.[21,23,42] as our main sources regarding notational conventions. The one loop gauge coupling constants and gaugino masses are then decoupled from the other parameters. Their scale dependence can be expressed explicitly,

$$\tilde{\alpha}_a(t) \equiv \frac{g_a^2(t)}{(4\pi)^2} = \frac{\tilde{\alpha}_a(0)}{1 - \beta_a t + \tilde{\alpha}_a(0)\Delta_a}, \quad M_a(t) = \frac{M_a(0)}{1-\beta_a t}, \tag{16}$$

where:

$$\beta_a \equiv b_a\tilde{\alpha}_a(0) = \frac{b_a g_a^2(0)}{(4\pi)^2}, \quad \Delta_a = \delta_a + \sum_i \tilde{b}'^i_a \log\left[(T_i+\bar{T}_i)|\eta(T_i)|^4\right]. \tag{17}$$

The string threshold corrections Δ_a include a moduli-independent part δ_a, to be neglected in the following, along with the familiar moduli-dependent part [9]. The β-function slope parameters b_a are defined as: $\partial g_a/\partial logQ = -b_a g_a^2/(4\pi)^2$, the values for the standard model group factors being: $b_a = (3, -1, -11)$, subject to the tree level normalization convention, $g_a^2(0)k_a = g_X^2$.

The renormalization group equations for the Yukawa coupling constants involve the gauge coupling constants and a dependence on generations through the Yukawa couplings. Motivated by the large hierarchy between third and first or second generations, we shall adopt here the approximation where all entries in $\lambda^x_{ij}, [x = u, d, e]$ are neglected relative to the $(3,3)$ entry. Denoting the corresponding (33) Yukawa matrix elements as $\lambda_t, \lambda_b, \lambda_\tau$,

the index standing for top- and bottom-quark and tau-lepton, respectively, one obtains the simplified third generation evolution equations [23]:

$$\begin{aligned}
\frac{d\tilde{Y}_t(t)}{dt} =& \tilde{Y}_t(t)\Big(y_{ua}\tilde{\alpha}_a(t) - 6\tilde{Y}_t(t) - \tilde{Y}_b(t)\Big), \\
\frac{d\tilde{Y}_b(t)}{dt} =& \tilde{Y}_b(t)\Big(y_{da}\tilde{\alpha}_a(t) - 6\tilde{Y}_b(t) - \tilde{Y}_t(t) - \tilde{Y}_\tau(t)\Big), \\
\frac{d\tilde{Y}_\tau(t)}{dt} =& \tilde{Y}_\tau(t)\Big(y_{ea}\tilde{\alpha}_a(t) - 4\tilde{Y}_\tau(t) - 3\tilde{Y}_b(t)\Big),
\end{aligned} \tag{18}$$

where $\tilde{Y}_x(t) = \frac{\lambda_x^2(t)}{(4\pi)^2}, [x = t, b, \tau]$ and $y_{ua} = (\frac{16}{3}, 3, \frac{13}{9}), y_{da} = (\frac{16}{3}, 3, \frac{7}{9}), y_{ea} = (0, 3, 3)$. An analytic solution to these equations exists in the approximation $\lambda_t \gg \lambda_b, \lambda_\tau$ [23],

$$\tilde{Y}_t(t) = \frac{\tilde{Y}_t(0)E_1(t)}{1+6\tilde{Y}_t(0)F_1(t)}, \quad \tilde{Y}_b(t) = \frac{\tilde{Y}_b(0)E_2(t)}{(1+6\tilde{Y}_t(0)F_1(t))^{\frac{1}{6}}}, \quad \tilde{Y}_\tau(t) = \tilde{Y}_\tau(0)E_3(t),$$

where:

$$E_1(t) = \prod_a \left(\frac{\alpha_a(t)}{\alpha_a(0)}\right)^{\frac{y_{ta}}{b_a}}, \quad F_1(t) = \int_0^t dt' E_1(t'),$$

$$E_2(t) = E_1(t)(1-\beta_1 t)^{\frac{2}{3b_1}}, \quad E_3(t) = (1-\beta_2 t)^{-\frac{3}{b_2}}(1-\beta_1 t)^{-\frac{3}{b_1}}. \tag{19}$$

The supersymmetry breaking parameters mix together as well as with the gauge and Yukawa parameters. For the $A^x_{ij}, [x = u, d, e]$, we shall adopt an analogous approximation to the above, namely, dropping all but the $(i, j) = (3, 3)$ entries. This approximation is justified by the fact that these quantities always enter through products, $A^x_{ij}\lambda^x_{ij}$. The simplified evolution equations for the $(3, 3)$ parameters, designated as A_t, A_b, A_τ, read [23]:

$$\begin{aligned}
\frac{dA_t(t)}{dt} =& y_{ua}\tilde{\alpha}_a(t)\frac{M_a(t)}{m_g} - 6\tilde{Y}_t(t)A_t(t) - \tilde{Y}_b(t)A_b(t), \\
\frac{dA_b(t)}{dt} =& y_{da}\tilde{\alpha}_a(t)\frac{M_a(t)}{m_g} - 6\tilde{Y}_b(t)A_b(t) - \tilde{Y}_t(t)A_t(t) - \tilde{Y}_\tau(t)A_\tau(t), \\
\frac{dA_\tau(t)}{dt} =& y_{ea}\tilde{\alpha}_a(t)\frac{M_a(t)}{m_g} - 3\tilde{Y}_b(t)A_b(t) - 4\tilde{Y}_\tau(t)A_\tau(t).
\end{aligned} \tag{20}$$

None of the other matrix elements, $A^x_{ij}, [x = u, d, e]$ with $(i, j) \neq (3, 3)$, are coupled to the $\lambda^x_{33}(t)$, so that these parameters have no other renormalization scale dependence in the present approximation except for that arising from gaugino loops. This affects the diagonal elements by a generation independent contribution analogous to that arising in eqs.(20). The approximate solution can be obtained by a simple quadrature,

$$A^x_{ij}(t) = A^x_{ij}(0) + y_{xa}\tilde{\alpha}_a(0)\frac{M_a(0)}{m_g}\frac{t}{1-\beta_a t}\delta_{ij} \quad [x = u, d, e; \quad (i, j) \neq (3, 3)].$$

The renormalization group equations for the scalars masses include contributions from gauginos loops as well those from scalars and Higgs bosons loops. In the above approximation of dominant $\lambda^x_{33}(t)$ Yukawa couplings, the scalars mass matrix elements, $(i,j) = (3,3)$, and $(i,3),(3,i),[i=1,2]$ decouple from other Yukawa matrix elements and have a scale evolution governed for the $(3,3)$ entries by the equations [21]:

$$\begin{aligned}
\frac{d(\tilde m^2_Q)_{33}}{dt} =& z_{Qa}\tilde\alpha_a M_a^2 - \tilde Y_t(\tilde m^2_Q + \tilde m^2_{Q^c} + \bar\mu_2^2 + m_g^2 A_t^2) - \tilde Y_b(\tilde m^2_Q + \tilde m^2_{b^c} + \bar\mu_1^2 + m_g^2 A_b^2),\\
\frac{d\tilde m^2_{Q^c}}{dt} =& z_{Q^c a}\tilde\alpha_a M_a^2 - 2\tilde Y_Q(\tilde m^2_Q + \tilde m^2_{Q^c} + \begin{pmatrix}\bar\mu_2^2\\ \bar\mu_1^2\end{pmatrix} + m_g^2 A_Q^2),\\
\frac{d\tilde m^2_L}{dt} =& z_{La}\tilde\alpha_a M_a^2 - \tilde Y_\tau(\tilde m^2_L + \tilde m^2_{\tau^c} + \bar\mu_1^2 + m_g^2 A_\tau^2),\\
\frac{d\tilde m^2_{L^c}}{dt} =& z_{L^c a}\tilde\alpha_a M_a^2 - 2\tilde Y_\tau(\tilde m^2_\tau + \tilde m^2_{\tau^c} + \bar\mu_1^2 + m_g^2 A_\tau^2),
\end{aligned} \tag{21}$$

and for the entries (i,j) with $[i=1,2, \quad j=3]$ or $[i=3, \quad j=1,2]$ by the equations,

$$\frac{d}{dt}(\tilde m^2_{Q,Q^c,L,L^c})_{ij} = -\left[\frac{1}{2}(\tilde Y_t + \tilde Y_b), \tilde Y_Q, \frac{1}{2}\tilde Y_\tau, \tilde Y_\tau\right](\tilde m^2_{Q,Q^c,L,L^c})_{ij}. \tag{22}$$

We have omitted writing the argument t for the various coupling constants and have denoted $Q=[t,b], Q^c=[t^c,b^c], L=[\tau,\nu_\tau], L^c=\tau^c$ with $z_{Qa}=4C_2(R_\alpha)$, corresponding to the quadratic Casimir operator in representation R_α for the standard model gauge group factor $a=3,2,1$, such that $z_{Qa}=(\frac{16}{3},3,\frac{1}{9}), z_{u^c a}=(\frac{16}{3},0,\frac{16}{9}), z_{d^c a}=(\frac{16}{3},0,\frac{4}{9}), z_{L^c a}=(0,3,1), z_{e^c a}=(0,0,4)$.

For the scalars masses matrix elements, (i,j) with $[i=1,2, j=1,2]$, we shall include the contributions of the Yukawa interactions perturbatively by retaining the terms with leading power of $t=\log\frac{M_X^2}{Q^2}$. The gauge interactions contributions, being generation independent, have the same form as those appearing in eq.(21), and are then included exactly. The relevant formulas read [21]:

$$\begin{aligned}
(\tilde m^2_Q(t))_{ij} \approx& (\tilde m^2_Q(0))_{ij} + \delta_{ij}G_{Qa}(t) + t\Bigg[\Big[-\frac{1}{2}\big(\{\lambda_u\lambda_u^\dagger, \tilde m^2_Q\}\big)_{ij} - (\lambda_u \tilde m^2_Q \lambda_u^\dagger)_{ij}\\
&-\bar\mu_2^2(\lambda_u\lambda_u^\dagger)_{ij} - m_g^2\Big((\lambda_u A_u)(\lambda_u A_u)^\dagger\Big)_{ij}\Big] + [\lambda_u\to\lambda_d, \bar\mu_2\to\bar\mu_1]\Bigg],\\
(\tilde m^2_{Q^c}(t))_{ij} \approx& (\tilde m^2_Q(0))_{ij} + \delta_{ij}G_{Q^c a}(t) + t\Bigg[-\big(\{\lambda_Q^\dagger\lambda_Q, \tilde m^2_{Q^c}\}\big)_{ij}\\
&-2(\lambda_Q^\dagger \tilde m^2_{Q^c}\lambda_Q)_{ij} - 2\begin{pmatrix}\bar\mu_2\\ \bar\mu_1^2\end{pmatrix}(\lambda_Q^\dagger\lambda_Q)_{ij} - 2m_g^2\Big((\lambda_Q A_Q)^\dagger((\lambda_Q A_Q)\Big)_{ij}\Bigg],
\end{aligned} \tag{23}$$

where we understand that terms on the right hand sides are evaluated at the electroweak scale and where

$$G_{Qa}(t) = \frac{1}{2}z_{Qa}\tilde\alpha_a(0)M_a^2(0)f_a(t), \quad [f_a(t) = \frac{(2-\beta_a t)t}{(1-\beta_a t)^2}]$$

stands for gaugino loop contributions. The slepton mass matrices $\tilde m^2_L, \tilde m^2_{L^c}$ are obtained by analogous formulas to those for $\tilde m^2_Q, \tilde m^2_{Q^c}$ above by substituting slepton mass matrix for squark mass matrices and letting $\lambda_d \to \lambda_e, \lambda_u \to 0$.

The mass parameters $\bar\mu_i^2 = \mu_i^2 - \mu^2$, appearing in formulas above for scalar masses, refer to the coupling constants in the Higgs bosons scalar potential,

$$V(H_i) = \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 - \mu_3^2(H_1H_2 + c.c.) + \frac{g_1^2+g_2^2}{8}(|H_1|^2 - |H_2|^2)^2, \tag{24}$$

where $\mu_i^2 = \tilde m^2_{H_i} + \mu^2 \quad [i=1,2], \quad \mu_3^2 = -B_\mu m_g \mu$. For completeness, we quote the scale evolution equations for these Higgs sector parameters, again in the simplified version in which our calculations are ferformed, namely, predominant $(3,3)$ trilinear couplings [23,42]:

$$\begin{aligned}
\frac{d\mu^2(t)}{dt} =& \mu^2\left[3\tilde\alpha_2 + \tilde\alpha_1 - 3(\tilde Y_t + \tilde Y_b + \frac{1}{3}\tilde Y_\tau)\right],\\
\frac{d\mu_1^2(t)}{dt} =& 3\tilde\alpha_2 M_2^2 + \tilde\alpha_1 M_1^2 + \left(3\tilde\alpha_2 + \tilde\alpha_1 - 3(\tilde Y_t + \tilde Y_b + \frac{1}{3}\tilde Y_\tau)\right)\mu^2 - 3\tilde Y_b(\tilde m_t^2 + m^2_{b^c} + \bar\mu_1^2 + m_g^2 A_b^2),\\
\frac{d\mu_2^2(t)}{dt} =& 3\tilde\alpha_2 M_2^2 + \tilde\alpha_1 M_1^2 + \left(3\tilde\alpha_2 + \tilde\alpha_1 - 3(\tilde Y_t + \tilde Y_b + \frac{1}{3}\tilde Y_\tau)\right)\mu^2 - 3\tilde Y_t(\tilde m_t^2 + m^2_{t^c} + \bar\mu_2^2 + m_g^2 A_t^2),\\
\frac{d\mu_3^2(t)}{dt} =& -(3\tilde\alpha_2 M_2 + \tilde\alpha_1 M_1 - 3m_g A_t \tilde Y_t)\mu + \frac{1}{2}(3\tilde\alpha_2 + \tilde\alpha_1 - 3\tilde Y_t - 3\tilde Y_b - \tilde Y_\tau)\mu_3^2.
\end{aligned} \tag{25}$$

Let us recall here that the weak hypercharge D-terms add an extra contribution in the scale evolution of scalars masses [22],

$$\frac{d\tilde m^2_\alpha}{dt} = -Y_\alpha \tilde\alpha_1(t)S(t) + \cdots, \quad \left[S(t) = \frac{S(0)}{1-\beta_1 t}\right],$$

$$S(t) = \sum_{gen}(\tilde m^2_Q - 2\tilde m^2_{U^c} + \tilde m^2_{D^c} - \tilde m^2_L + \tilde m^2_{E^c}) - m^2_{H_1} + m^2_{H_2} = Trace(\tilde m^2_\alpha Y_\alpha),$$

where $Y_\alpha = (\frac{1}{6}, -\frac{2}{3}, \frac{1}{3}, -\frac{1}{2}, 1, -\frac{1}{2}, \frac{1}{2})$ is the hypercharge of the low energy fields in the order
$(Q, U^c, D^c, L, E^c, H_1, H_2)$, and dots refer to gauge and Yukawa terms quoted above. This term contributes only in cases with non-universal boundary conditions [22]. Although it may significantly influence the spectra, it was found in ref.[33] to affect negligibly flavor changing observables and so will be neglected in the sequel.

At the electroweak breaking scale, which we shall identify with the Z-boson mass, $Q \approx m_Z, \quad t_Z = 2\log\frac{M_X}{m_Z}$, scalars masses obtain additional contributions from Yukawa couplings F-terms and gauge couplings D-terms. Let us adopt the familiar notation in which the three generations of left chiral squarks (L) and anti-squarks (R) are combined together into a single six-dimensional coloumn vector, so that the scalars mass terms in the Lagrangian appear in the 6×6 matrix form,

$$-L_{mass} = \sum_{Q=U,D}(\tilde Q_L, \tilde Q_R^\dagger)\begin{pmatrix} M^{Q2}_{LL} & M^{Q2}_{LR} \\ M^{Q2\dagger}_{LR} & M^{Q2}_{RR}\end{pmatrix}\begin{pmatrix}\tilde Q_L^\dagger \\ \tilde Q_R\end{pmatrix} + [Q \to L], \tag{26}$$

The full squarks and sleptons masses, comprising the above supersymmetry breaking contributions evolved from the the string scale down to the electroweak scale, along with those induced at the electroweak scale, read [42]:

$$\begin{aligned}
(\tilde M^{Q2}_{LL})_{ij}(t_Z) =&(\tilde m^2_Q)_{ij}(t_Z)+(M_Q M_Q^\dagger)_{ij}\mp m_Z^2\cos 2\beta\big(\frac12-\frac13\binom{2}{1}\sin^2\theta_W\big)\delta_{ij},\\
(\tilde M^{Q2}_{RR})_{ij}(t_Z) =&(\tilde m^2_{Q^c})_{ij}(t_Z)+(M_Q^\dagger M_Q)_{ij}\mp m_Z^2\cos 2\beta\frac13\binom{2}{1}\sin^2\theta_W\delta_{ij};\\
(\tilde M^{L2}_{LL})_{ij}(t_Z) =&(\tilde m^2_L)_{ij}(t_Z)+(M_L M_L^\dagger)_{ij}\pm \frac12 m_Z^2\cos 2\beta\begin{pmatrix}1-2\sin^2\theta_W\\ 1\end{pmatrix}\delta_{ij},\\
(\tilde M^{L2}_{RR})_{ij}(t_Z) =&(\tilde m^2_{L^c})_{ij}(t_Z)+(M_L^\dagger M_L)_{ij}- m_Z^2\cos 2\beta\sin^2\theta_W\delta_{ij};
\end{aligned} \tag{27.a}$$

$$(\tilde M^{u2}_{LR})_{ij}=\big[m_g A^u_{ij}+\frac{\mu}{\tan\beta}\big](M_u)_{ij},\quad (\tilde M^{(d,e)2}_{LR})_{ij}=\big[m_g A^{(d,e)}_{ij}+\mu\tan\beta\big](M_{(d,e)})_{ij}; \tag{27.b}$$

the upper and lower positions in eqs.(27.a) referring to the up-squark and down-squark cases, respectively, or to slepton and sneutrino cases, respectively. We denote by $M_x, [x=u,d,e]$ the quark and lepton mass matrices, $M_u=\lambda_u(t_Z)\frac{v_2}{\sqrt2}, \quad M_{d,e}=\lambda_{d,e}(t_Z)\frac{v_1}{\sqrt2}$, where $< H^0_i >= v_i/\sqrt2$, such that $v_1^2+v_2^2=v^2$, $\tan\beta=v_2/v_1$, $m_Z^2=(g_1^2+g_2^2)v^2/4$.

The condition of radiative electroweak symmetry breaking, as derived by minimizing the Higgs bosons potential, eq.(24), involves solving the two equations [23,42]:

$$\tan^2\beta=\frac{\mu_1^2(t_Z)+m_Z^2/2}{\mu_2^2(t_Z)+m_Z^2/2},\qquad \sin 2\beta=\frac{2\mu_3^2(t_Z)}{\mu_1^2(t_Z)+\mu_2^2(t_Z)}. \tag{28}$$

For completeness, we also quote the mass terms of charginos and neutralinos, $L_{mass}=-[\chi^{-\dagger}M_c\chi^+ + c.c.]-\frac12\chi^{0T}M_n\chi^0$, with

$$M_c=\begin{pmatrix} M_2 & \frac{g_2v_2}{\sqrt2}\\ \frac{g_2v_1}{\sqrt2} & -\mu\end{pmatrix},\quad M_n=\begin{pmatrix} M_1 & 0 & -\frac{g_1v_1}{2} & \frac{g_1v_2}{2}\\ & M_2 & \frac{g_2v_1}{2} & -\frac{g_2v_2}{2}\\ & & 0 & \mu\\ & & & 0\end{pmatrix}.$$

referring to the basis, $\chi^\pm=(-i\tilde W^\pm,\tilde H^\pm_{2,1}),\quad \chi^0=(-i\tilde B^0,-i\tilde W^0,\tilde H^0_1,\tilde H^0_2)$. The contributions of super-box or super-penguin diagrams to flavor changing neutral current processes, with external quarks and leptons, are most conveniently described (in the so-called insertion approximation) in terms of generation mixing terms for scalars mass matrices in the following representation:

$$\tilde M'^2_{MN}=V_M\tilde M^2_{MN}V_N^\dagger,\quad [M,N=L,R]$$

corresponding to the super-CKM mass bases for the quarks and leptons superfields with generation diagonal D-terms Yukawa interactions, $\tilde Q_i^\star T^a\lambda_a q_j\delta_{ij}+\tilde L_i^\star T^a\lambda_a e_j\delta_{ij}+c.c.$. The transformation matrices $V^q_M\quad [M=L,R; q=u,d,e]$, relating current and mass bases for quarks and leptons, are defined in the conventional way so as to satisfy, $M_q=V_L^{q\dagger}(M_q)_{diag}V^q_R\quad [q=$

$u, d, e]$. The above notation emphasizes the important fact that flavor changing observables are sensitive to supersymmetry breaking as well as to Yukawa interactions. Note that only the relative product, identified with the (Cabibbo-Kobayashi-Maskawa) CKM matrix $V = V_{CKM} = V_L^u V_L^{d\dagger}$, is physically observable if one restricts to fermions weak interactions, while additional information on the structure of the fermions mass matrices is needed to describe the physical flavor changing paramaters.

2.4 Yukawa coupling matrices

To obtain boundary conditions for the Yukawa coupling constants at the string unification scale, one might use, as in the so-called bottom-up approach, the available experimental information for quarks and leptons masses and weak mixing angles to determine the Yukawa coupling matrices at the electroweak scale and next evolve these up to the string unification scale by integrating the renormalization group equations [21]. Here, we shall follow a simpler approximate procedure: We assume some fixed choice for the Yukawa coupling matrices at the unification scale, with a structure consistent with orbifolds predictions [39,43], and deduce from this the implied form of V_M^q at electroweak scale. The analysis of Casas and Muñoz [43] of the generation dependence of Yukawa matrices for orbifolds favors the following structure,

$$\lambda^q(M_X) = \begin{pmatrix} 0 & a & 0 \\ a & A & c_q \\ 0 & c_q & B \end{pmatrix}, \quad [q = u, d, e] \tag{29}$$

the entries here being real numbers, a choice which can always be achieved by performing suitable chiral transformation field redefintions. The choice of eq.(29) has a suppressed value for the first generation, assumed to arise from non-renormalizable interactions, and a nearly diagonal structure for the second and third generations couplings, assumed to arise from renormalizable couplings, the suppression of off-diagonal terms being caused by a proper choice of widely spaced fixed points. The assumed hierarchy of parameters in eq.(29) is: $B \gg A \gg a, c_q$. For top-quark masses, $m_t \leq 100$ GeV, the scale evolution of the Yukawa matrix is dominated by gauge bosons loops and can be represented approximately as an overall scaling factor determined by the gauge coupling constants,

$$M_q(t) = F_q(t) M_q(0), \quad F_q(t) = \prod_a \Big(\frac{\alpha_a(t)}{\alpha_a(0)}\Big)^{y_a/b_a}.$$

Full formulas for the scale evolution factor are quoted in ref.[43]. For larger top-quark masses, the top-quark Yukawa couplings compete with gauge interactions and affect mainly the matrix elements, λ_{ij}^x with $i = 3$ or $j = 3$, corresponding here to the parameters $B(t)$ and $c_q(t)$. Considering, for definiteness, the down-quarks case, the mass eigenvalues for a Yukawa matrix of the form of eq.(29) are: $m_d \approx a^2/A, m_s \approx A, m_b \approx B$, so that only m_b is scale-dependent in the approximation of dropping Yukawa interactions in the scale evolution, while c_q is scale dependent if one includes third generation couplings. For symmetric mass matrices, as is the case here, left and right transformation matrices are equal, $V_L^d = V_R^d$, and given by:

$$V_L^d = V_R^d \approx \begin{pmatrix} 1 & -\sqrt{\frac{m_d}{m_s}} & \frac{\sqrt{m_d m_s} c_d}{m_b m_s} \\ \sqrt{\frac{m_d}{m_s}} & 1 & -\frac{c_d}{m_b} \\ \frac{\sqrt{m_d m_s} c_d}{m_b^2} & \frac{c_d}{m_b} & 1 \end{pmatrix}, \tag{30}$$

so that the transformation matrices are fully specified, up to a single unknown parameter, $c_d = c_d(t_Z)$ [43]. Note that $(M_d)_{diag} = (-m_d, m_s, m_b)$, and that the negative mass eigenvalue can always be absorbed, if so desired, by a chiral transformation, say, $d_L \to -d_L, d_R \to d_R$. We have only quoted above explicit formulas for the down-quarks case. We shall assume that similar structures hold for up-quarks and leptons mass matrices, so one obtains for these cases analogous formulas by replacing $(d, s, b) \to (u, c, t)$ or (e, μ, τ) with two additional free parameters, $c_u(t), c_e(t)$. The values of these parameters at the electroweak scale can be obtained by identifying $V_L^u V_L^{d\dagger}$ with the CKM-matrix, which supplies four relations expressing $c_u(t), c_d(t)$ as linear combinations of the CKM matrix elements, V_{ub}, V_{cb} or V_{td}, V_{ts}. We shall use in our calculations the above form of Yukawa couplings for up and down quarks and for leptons, and following ref.[43], tentatively set the $c_q(t)$ parameters as follows: $c_u(t_Z) \approx m_s\sqrt{2}, c_d(t_Z) \approx m_c, c_l(t_Z) = m_\mu$. Of course, other choices for the Yukawa matrices will lead to alternative expressions for the transformation matrices. For symmetric Yukawa matrices, under restrictive assumptions on the number of texture zeros, the number of choices is known to be limited to only five cases [44].

3. Results and discussion

In this section we shall present numerical results for the dimensionless parameters defined by:

$$\delta_{MN} = \frac{\tilde{M}'^2_{MN}}{\tilde{m}^2} = \frac{V_M^q \tilde{M}^{q2}_{MN} V_N^{q\dagger}}{\tilde{m}^2}, \tag{31}$$

where the mass factor in the denominator $\tilde{m}$ represents some average scalar superpartner mass, which we shall set for the equal chirality cases δ_{MM} to the weighted trace of the mass submatrix of fixed chirality $M = L, R$ and for the mixed chirality case δ_{LR} to the weighted average of the full mass matrix trace.

3.1 Inputs

The basic parameters entering the calculations are set as follows:

(a) Masses: $m_Z = 91.17GeV, v = 246GeV; m_t = 170GeV, m_b = 5.6GeV, m_\tau = 1.784GeV, m_c = 1.35GeV, m_s = 0.175GeV, m_\mu = 0.1056GeV, m_u = 5.1MeV, m_d = 8.9MeV, m_e = 0.51MeV.$

(b) Gauge coupling constants : $g_1^2(m_Z) = 0.127, g_2^2(m_Z) = 0.425, g_3^2(m_Z) = 1.44, g_X = \frac{1}{\sqrt{2}}, M_X = 5.27 g_X 10^{17}GeV = 3.73 10^{17}GeV$, with $t_Z = 2\log\frac{M_X}{m_Z} = 71.9$. The boundary conditions $\alpha_a(0)$ are evaluated via eq.(16).

The dilaton VEV is set at $< S >= \frac{1}{g_X^2} = 2$. The numerical values for the moduli VEVs will be chosen in the various cases to be considered below on the basis of fits to gauge coupling constants unification, and specifically to $\alpha_s(t_Z)$ and $\sin^2\theta_W(t_Z)$ [8,45]. Recall that the radiative corrections to these quantities involve the linear combinations, $A'^i = \frac{k_2}{k_1} b_1'^i - b_2'^i, \quad B'^i = b_1'^i + b_2'^i - \frac{k_1+k_2}{k_3} b_3'^i$, so defined as to be free of the δ^i_{GS}. Note that the typical values of order $T \approx 10$ that are found in these fits [8] lie an order of magnitude above those found in mimimizing the scalar potential in gaugino condensation models [17]. In the present work we shall not consider CP violation effects and therefore will set to zero the imaginary parts of dilaton and moduli VEVs and phases of auxiliary fields, $\alpha_S = \alpha_T = 0$. We also shall neglect contributions to supersymmetry breaking parameters, eq.(15), involving derivatives with respect to dilaton and moduli fields of the Yukawa coupling constants.

The Green-Schwarz parameters δ^i_{GS}, representing gauge group independent contribu-

tions to duality anomalies cancellation, are calculable for orbifolds on a model-by-model basis via the β-function slopes of $N=2$ suborbifolds with i-plane fixed. The typical values in a minimal standard model orbifold can be calculated from formulas given at the end of Subsection 2.1. One finds for the diagonal sums, $\delta_{GS}=\sum_i \delta^i_{GS}$, values of positive sign inside the interval, $\delta_{GS}\approx[5-10]$. (Note that our conventions for the parameters $b_a, b'^i_a, \delta^i_{GS}$ differ by a sign from those of ref.[19].) This contrasts with the range $\delta_{GS}\approx[25-50]$, favored by gaugino condensation models [17].

The parameters b'^i_a also are model dependent. If large moduli VEVs are used, then, because of the exponential dependence on T_i, this strongly constrains the $\tilde b'^i_a$ of overall unrotated planes if one is to avoid excessive threshold corrections. Indeed, expressing these corrections to gauge coupling constants in terms of an effective string unification scale M'_X, one finds:

$$\frac{M'_X}{M_X}=\prod_i\left[(T_i+\bar T_i)|\eta(T_i)|^4\right]^{-\frac{\tilde b'^i_a}{2b^i_a}}\approx\prod_i(T_i+\bar T_i)^{\frac{\tilde b'^i_a}{2b^i_a}}e^{\frac{\pi T_i\tilde b'^i_a}{6b^i_a}}, \tag{32}$$

where the large radius limit was used in the second step. We see that a reduction of order 10 in the string scale requires for the exponent $-\frac{\tilde b'^i_a}{2b_a}$ a positive value of absolute magnitude close to unity. An analogous constraint applies for the more relevant exponent, $-\frac{\tilde b'^i_a k_k-\tilde b'^i_b k_a}{2(b_a k_b-b_b k_a)}$ associated to the unification of two group factors, G_a and G_b.

The experimental information for δ_{MN} can be obtained from flavor changing neutral current observables involving mass differences in the neutral mesons systems $(K-\bar K, B-\bar B, D-\bar D)$ and branching ratios of transitions such as $q_i\to q_j+\gamma$ or $l_i\to l_j+\gamma, \quad [i\neq j]$. Upper bounds are derived by assuming that these observables are solely accounted in terms of contributions of superbox or superpenguin diagrams with gluino and squark, or photino and slepton, exchanges. Denoting the average superpartner scalar mass by $\tilde m$, the gluino and minimal neutralino masses by M_3 or $M_{\tilde\gamma}$, then the calculated mass differences $\delta m_K, \delta m_B, \delta m_D$ and branching ratios B have a dependence on superpartners masses which can be separated into two variables: $\tilde m$ and $x_{3,\gamma}=\frac{M^2_{3,\gamma}}{\tilde m^2}$. A study of formulas quoted in refs.[20,34] reveals that $\delta m_{K,B}\propto\frac{\delta^2_{MN}}{\tilde m^2}F_B(x)$, with $F_B(x)$ smooth functions of x, so that the box diagrams bounds imply the scaling law, $\delta_{MN}\propto\tilde m, \quad [M,N=L,R]$. Using $x=1$ as a reference value, then account of the $x-$dependence results in stricter (weaker) bounds depending on $x<1(x>1)$. The penguin diagrams contributions to branching ratios, neglecting the contributions of $A-$ and $\mu-$ coupling terms [20], read: $B\propto\delta^2_{MM}F_{P1}(x)/\tilde m^4$ or $\delta^2_{LR}F_{P2}(x)/\tilde m^2$, which implies the scaling laws, $\delta_{MM}\propto\tilde m^2$, $\delta_{LR}\propto\tilde m$, times smooth functions of x.

The numerical values of presently known bounds for δ_{MN}, as obtained from results in refs.[20,27,34], are quoted in Table 1. More stringent bounds are found for the quantities in Table 1 designated as, $\delta_{ij}=(\delta_{LL}\delta_{RR})^{\frac12}$, which correspond to contributions implying simultaneously non-vanishing chiral left and right mass terms, $\delta_{LL}\delta_{RR}\neq 0$. A similar situation holds for the case of simultaneously non-vanishing $(\delta_{LR})_{ij}$ and $(\delta_{LR})_{ji}$ matrices. The bounds in Table 1 for δ_{LR} correspond precisely to the geometric average of these matrix elements.

3.2 General features of model

It is useful to identify the various sources of off-diagonal contributions to the matrices $(\delta_{MM})_{ij}$. The F- and D-terms (second and third terms in the formulas of eq.(27.a)) yield purely diagonal contributions. The solutions of the renormalization group equations (first terms in eq.(27.a)) give contributions the scalars mass matrices with the approximate structure, $(\tilde M^{Q2}_{LL})_{ij}(t) \approx (\tilde m^{Q2})_{ij}(0) + G_Q(t_Z)\delta_{ij} + \sum_{q'=u,d} y^{(q)}_{q'} (M_q M^\dagger_{q'})_{ij}$, under the simplifying assumption that the contributions from boundary conditions, gauginos couplings and Yukawa couplings add up linearly. Off-diagonal matrix elements cannot arise from the family independent gauginos couplings. They also cannot arise from the boundary conditions mass matrices if these feature family universality ($\tilde m^2_{ij}$ is multiple of identity) or alignment with the fermions mass matrices ($V_M \tilde m^2_{MN}(0) V^\dagger_N$ are diagonal matrices). As is also well known [20], the radiatively induced off-diagonal terms from Yukawa couplings are numerically significant only for the submatrix $\tilde M^{d2}_{LL}$, yielding for flavor changing parameters the approximate result:

$$(\delta^d_{LL})_{ij} \approx \frac{y^{(d)}_u}{\tilde m^2_d} \Big(V^\dagger (M^2_u)_{diag} V \Big)_{ij} \approx y^{(d)}_u \frac{m^2_t}{\tilde m^2_d} V^\dagger_{i3} V_{3j}, \tag{33}$$

with $y^{(d)}_u$ of order unity and $V = V_{CKM}$. (The chirality L is singled out because only left-chirality scalars of given charge interact with scalars of both charges and the charge d because of the predominant top-quark Yukawa coupling.) For these flavor changing terms of standard model origin one expects then important suppression due to small CKM mixing angles. Using numerical estimates quoted in ref.[34] for $y^{(d)}_u$, gives: $\delta^d_{12} \approx 7.3 \times 10^{-4}, \delta^d_{13} \approx 1.4 \times 10^{-3}$.

For the mixed chirality parameters, the relevant terms in the interaction basis which contribute to off-diagonal elements of $(\delta_{LR})_{ij}$ can be written approximately as:

$$(\tilde M^{2x}_{LR})_{ij} \approx m_g A^x_{ij}(t_Z) M^x_{ij}(t_Z) \approx M^x_{ij}(t_Z) \Big[-m_g \cos\theta (n_i + n_j) + M_a(0) y_{xa} \tilde\alpha_a(0) \delta_{ij} \frac{t}{1-\beta_a t} \Big]. \tag{34}$$

This formula clearly illustrates the fact that non-universal contributions (first term) could be masked by gaugino contributions (second term),in the presence of large gauginos masses. Because of the dominant $(3,3)$ entries in the fermions mass matrices, the expected pattern is: $(\delta^x_{LR})_{23} > (\delta^x_{LR})_{13} > (\delta^x_{LR})_{12}$.

Returning now to the unmixed chirality case, a simple estimate of the generation non-universality of boundary conditions can be performed by including the contributions of gaugino loops in the scale evolution only, while dropping entirely those of Yukawa couplings. The physical mass matrices in the weak interaction basis are then diagonal ones and given by:

$$(\tilde M^{x2})_{ij}(t_Z) = m^2_g \Big(1 + n_{xi} \cos^2\theta + \frac{3}{2} \sin^2\theta \sum_a z_{xa} \tilde\alpha_a(0) f_a(t_Z) \Big) \delta_{ij}, \quad [x = Q, e] \tag{35}$$

using self-explanatory notations, with $f_a(t)$ defined at eq.(23) and dropping momentarily the suffix $MM = LL, RR$. Performing the transformation to the super-CKM basis under the simplifying assumption of negligible off-diagonal matrix elements $\tilde M^2_{ij} \approx 0 \quad [i \neq j]$, one obtains the approximate formulas:

$$\tilde M'^{x2}_{ij} = (V^x_M \tilde M^{x2} V^{x\dagger}_M)_{ij} \approx \tilde M^{x2}_{ii} V^{x\dagger}_{Mij} + \tilde M^{x2}_{jj} V^x_{Mij} + (\tilde M^{x2}_{33} - \tilde M^{x2}_{11}) V^x_{Mi3} V^{x\dagger}_{M3j} + \cdots, \quad [x = Q, e]$$

where dots refer to contributions proportional to $(\tilde{M}^{x2}_{ii} - \tilde{M}^{x2}_{22})$. Given the structure of V_M, eq.(30), the expected pattern for the flavor changing contributions associated with the boundary conditions is: $\delta_{12} \approx \delta_{23} > \delta_{13}$. Neglecting the F- and D-terms, one obtains for $(i,j) = (1,2)$ or $(1,3)$ the rough estimate:

$$\delta^x_{ij} \approx 2(V^{x\dagger}_M)_{ij} \frac{\tilde{M}^{x2}_{ii} - \tilde{M}^{x2}_{jj}}{\tilde{M}^{x2}_{ii} + \tilde{M}^{x2}_{jj}} \approx \frac{\cos^2\theta}{19} (V^{x\dagger}_M)_{ij} \frac{n_{xi} - n_{xj}}{1 + \frac{1}{19}\left(\frac{n_{xi}+n_{xj}}{2} - 18\right)\cos^2\theta}, \tag{36}$$

which shows that gauginos can have a strong dilution effect on non-universality at the unification scale, to the extent that the mixing angles θ lie not too close to 0 or π.

We shall now focus on the right chirality d-squarks case since this possesses simplifying features allowing to find upper bounds on the flavour changing parameters. Only the diagonal matrix elements here get renormalized by gauginos loops. Since this is proportional to the identity matrix it follows that the right chirality d-squark mass matrix has non-renormalized non-diagonal terms. Let us write down this matrix at the compactification scale in the basis where the d-quark Yukawa couplings are diagonal:

$$\tilde{M}'^{Q2}_{RRij} = \frac{m_0^2}{3}\left(\delta_{ij} + \mathrm{cotan}^2\theta \sum_k V^d_{Rik}(n_k+1)V^{d\dagger}_{Rkj}\right), \tag{37}$$

where $m_0 = M_a(0)$ is the common gaugino mass at unification scale and we denote, for notational convenience, $n_i = n_{Di}$. Defining the renormalized average mass by means of a numerical estimate for the gaugino loops contributions,

$$\tilde{m}^2_{av} = \frac{m_0^2}{3}(18 + \frac{n_1+n_2+n_3+3}{3}\mathrm{cotan}^2\theta), \tag{38}$$

we find the flavor changing parameters,

$$(\delta^d_{RR})_{ij} = \frac{\sum_k V^d_{ik} n_k V^{d\dagger}_{kj}}{19\tan^2\theta + \frac{(n_1+n_2+n_3+3)}{3}}. \tag{39}$$

This ratio explicitly depends on the arrangement of modular weights. Using the ansatz for the V^d matrix, eq.(30), the numerator can be evaluated. As we are looking for an upper bound we choose the arrangement generating the largest contribution. We also safely neglect the third family angle compared to the Cabibbo angle. Using unitarity the result can solely be expressed as a function of modular weight differences. Moreover the θ angle can be constrained by requiring that the scalar masses are positive at the compactification level. This implies that $\tan^2\theta > -\frac{1}{(n_m+1)}$, where n_m is the lowest modular weight. This yields the upper bounds:

$$\begin{aligned}
(\delta^d_{RR})_{12} &\leq \sqrt{\frac{m_d}{m_s}} \frac{\Delta n_1}{-19(n_m+1) + \frac{\Delta n_1 + \Delta n_2}{3}}, \\
(\delta^d_{RR})_{13} &\leq \frac{\sqrt{2 m_d m_s}}{m_b} \frac{\Delta n_2}{-19(n_m+1) + \frac{\Delta n_1 + \Delta n_2}{3}} \\
(\delta^d_{RR})_{23} &\leq \sqrt{2}\frac{m_s}{m_b} \frac{\Delta n_2}{-19(n_m+1) + \frac{\Delta n_1 + \Delta n_2}{3}},
\end{aligned} \tag{40}$$

where Δn_1 is the modular weight difference involving n_1, e.g., $\Delta n_1 = n_1 - n_3$ if $n_m = n_3$, and similarly for Δn_2. Notice that the order of magnitude of these mass insertions is given by the ratio of d-quark masses. Effectively these ratios are not very different from the elements of the CKM matrix. Indeed the ratios of the u-quark masses composing the u-quark Yukawa coupling diagonalising matrix are negligible compared with their d-quark counterparts. Thence $(\delta^d_{RR})_{12}$ is proportional to a very good approximation to $\sin\theta_C$. Numerical values can be extracted from eq.(40). For instance, using $n_1 = -2,\ n_2 = -3,\ n_3 = -1$, one obtains $(\delta^d_{RR})_{12} \leq 6.10^{-3},\ (\delta^d_{RR})_{13} \leq 2.5\ 10^{-4},\ (\delta^d_{RR})_{23} \leq 1.1\ 10^{-3}$. These analytic bounds are modified numerically by the subleading Yukawa couplings. Nevertheless their orders of magnitude is retrieved.

3.3 Non-universal modular weights assignments

Most examples of solutions quoted by Ibáñez and Lüst [8] are generation independent. It is clear, however, that relaxing the generation independence constraint could only increase the number of solutions. Rather than repeating the type of analysis in ref.[8] to seek for generic features of solutions involving a generation dependence, we choose here to consider three representative solutions, taken among the examples quoted in ref.[8], and consider generation dependent discrete perturbations of modular weights by a few units. We do not require consistency with anomalies cancellation and unification constraints. In all cases discussed below, we set the standard model Kac-Moody levels at: $k_a = (1, 1, \frac{5}{3})$. The following three cases will be considered:

(1) Case I: (ref.[45] and ref.[8], eq.(4.6)) This refers to a general, unspecified orbifold case with one modulus, constrained by gauge unification only and by a restricted interval for modular weights : $-3 \leq n_\alpha \leq -1$. It can be realized in terms of an isotropic orbifold or an anisotropic one with $T_1 \gg T_2, T_3$, both cases with all planes overall unrotated, $N_{unr} = 3$, or with two rotated planes. The generation independent isotropic case, disregarding therefore the anomalies cancellation condition, admits an unique solution consistent with the unification conditions on the strong and weak angle coupling constants [45]: $n_Q = n_D = -1, n_U = -2, n_L = n_E = -3, n_{H_1} + n_{H_2} = -5, -4$, with $Re(T) \approx 16$. (The suffices U, D, E here obviously refer to the corresponding antiparticles.) We choose here, $n_{H_1} = -2, n_{H_2} = -3$. Using the above values of modular weights yields: $b'_a = (6, 8, 18)$. The exponential dependence of threshold corrections and large VEV of the overall modulus entail a careful tuning of δ_{GS}. We shall set $\delta_{GS} = 7$, i.e., $\tilde{b}'^{\,i}_a = (-1, 1, \frac{19}{3}$, which gives $\frac{M'_X}{M_X} \approx (10^{-1}, 10^{-3}, 10^{-2})$ for $[a = 3, 2, 1]$. We shall implement generation dependence in terms of two subcases: (i) Case I-a: $n_{\alpha i} = n_\alpha - 1, n_{Li} = n_L + 1, n_{Ei} = n_E + 1, [i = 1, 2]$ and $n_{\alpha 3} = n_\alpha, n_{L3} = n_L, n_{E3} = n_E$ with $\alpha = Q, U^c, D^c$; (ii) Case I-b: Same assignments with $Re(T) = 1.3$.

(2) Case II: (ref.[8], Section 4) Z'_8 orbifold with two rotated planes, $i = 2, 3$, consistent with unification, $Re(T_1) \approx 24$ and anomalies cancellation, $\tilde{b}'^{2,3}_a = 0$. This admits several generation dependent solution, of which an example is: $n^i_{Q123} = (0, -1, 0), n^i_{D123} = (-1, 0, 0), n^i_{U1} = -\frac{1}{2}(0, 1, 1), n^i_{U23} = -\frac{1}{8}(6, 15, 3), n^i_{L1} = -\frac{1}{8}(14, 3, 7), n^i_{L23} = -\frac{1}{8}(14, 7, 3), n^i_{E1} = -(1, 0, 0), n^i_{E23} = -\frac{1}{8}(6, 15, 3), n^i_{H1} = -\frac{1}{4}(4, 3, 3), n^i_{H2} = -\frac{1}{8}(14, 3, 7)$. The β-functions slopes are calculated to be: $b'^{1}_a = (\frac{3}{2}, \frac{5}{2}, \frac{15}{2}), \quad b'^{i}_a = k_a \delta^i_{GS} = \frac{1}{4}\binom{29}{-7}$, for $i = \binom{2}{3}$. We shall choose $\delta^1_{GS} = \frac{5}{2}$, which gives $\delta_{GS} = 8$. The anisotropic torus with $T_1 \gg T_2, T_3, \quad e_2, e_3 \ll e_1 \approx 1$ has the modified weights, $n''_\alpha \approx \sqrt{3} n'_\alpha$, defined at eq.(15), given as: $n''_{Q123} = 0, n''_{D123} = -3, n''_{U123} = (0, -\frac{9}{4}, -\frac{9}{4}), n''_{L123} = -\frac{21}{4}, n''_{E123} = (-3, -\frac{9}{4}, -\frac{9}{4}), n''_{H_1} = -\frac{3}{2}, n''_{H_2} = -\frac{21}{4}$. We see that these effective weights cover a wider range than in the isotropic case.

We consider two versions based on this example: (i) Case II-a: Isotropic torus with overall weights: $n_{Q123} = n_{D123} = -1, n_{U1} = -1, n_{U2,3} = -3, n_{L123} = -3, n_{E1} = -1, n_{E2,3} = -3, n_{H_1} = -2, n_{H_2} = -3$. (ii) Case II-b: Anisotropic torus with $T_1 \gg T_2, T_3, \quad e_2, e_3 \ll e_1 \approx 1$. To simplify calculations we represent this case by using the usual weights, but with values covering a wide interval of the type described above. We use: $n_{Q123} = n_{D123} = n_{L123} = (0, -2, -3), n_{U123} = n_{E123} = (-1, -3, -3), n_{H_1} = -2, n_{H_2} = -3$.

(3) Case III: (ref.[8], Section 4) Z_6' orbifold with one rotated plane, $i = 3$, consistent with unification, $Re(T_1) = Re(T_2) \approx 10$, and anomalies cancellation. The first condition yields $\sum_{i=1,2} b_a^{'i} = (5, 4, -\frac{14}{3})$ and the second yields $b_a^{'3} = k_a \delta_{GS}^3 = -2$. To achieve satisfactory gauge coupling constants unification we shall choose: $\sum_{i=1,2} \delta_{GS}^i = 6$. Starting with the universal solution of ref.[8], $n_Q = n_D = n_U = n_L = n_E = -1, n_{H_1} = n_{H_2} = -1$, generation non-universality is implemented by considering two subcases. (i) Case III-a: $Re(T_1) = 10$, $n_{\alpha 1} = n_\alpha, n_{\alpha 2} = n_\alpha - 1, n_{\alpha 3} = n_\alpha - 2$; (ii) Case III-b: $n_{\alpha 12} = -1, n_{\alpha 3} = -2, \quad \alpha = [Q, U^c, D^c, L, E^c]$.

3.4 Electroweak breaking, soft parameters and low energy spectrum

We choose the dilaton-moduli mixing angle θ and gravitino mass m_g as our sole free parameters and determine the remaining parameters, namely, $\tan\beta$ and μ, by solving eqs.(28) at the electroweak scale with the solutions of the renormalization group equations for $\mu_i^2(t), \quad [i = 1, 2, 3]$ obtained by integrating the coupled equations (20), (21), (25), using eq.(19) for Yukawa couplings. We shall present detailed results for the following choice of supersymmetry breaking free parameters: $m_g = 200$ GeV, $\theta \in [0, \pi]$ and for Yukawa coupling constant : $\lambda_t(t_Z) = 0.85$. Solving the minimization equations (28) to find $\tan\beta$ and μ, will then determine all remaining parameters of the model. Let us first try to assess the accuracy of the approximation $\lambda_t \gg \lambda_b, \lambda_\tau$ that we shall use. This is expected to be a reliable one provided $\tan\beta$ is of order unity. Thus, starting, say, with $\tan\beta = 5.5$ and $\tilde{Y}_x(t_Z) = [5.810^{-3}, 2.010^{-4}, 6.410^{-5}], \quad [x = u, d, e]$ and integrating upwards to large scales in the above approximation, yields: $\tilde{Y}_x(0) = [3.710^{-4}, 1.210^{-5}, 7.710^{-6}]$. Integrating now downwards from M_X with the exact evolution equations, yields: $\tilde{Y}_x(t_Z) = [3.110^{-3}, 1.610^{-4}, 2.210^{-5}]$, which is in fair agreement with the starting values.

The numerical applications presented in this work should usefully complement those of ref.[19]. The Calabi-Yau and O-II (orbifolds with small threshold corrections) scenarios have no impact on the flavor changing neutral current issue. Building up on the (large threshold correction) O-I scenario of ref.[19], we consider here a wide variety of non-universal modular weights and also include the b-quark and τ-lepton Yukawa couplings. The latter item opens up the possibility of independently fixing $\tan\beta$ when solving for the radiative electroweak breaking condition. However, the approximation $\lambda_t \gg \lambda_b, \lambda_\tau$, eq.(19), that we use is a limiting factor here, because this weakens the sensitivity to these coupling constants of the electroweak symmetry breaking equations. We should mention at this point that our method of solving for $\tan\beta$ is not very robust. We proceed numerically along the following steps: First, we select a trial value for $\tan\beta$, which fixes $\lambda_b(t_Z), \lambda_\tau(t_Z)$. Second, we evaluate all other parameters by solving the renormalization group equations for a discrete set of values for $\frac{\mu(0)}{m_g}$ inside the interval $[1 - 4]$. Finally, we solve for $\mu(0)$ the combination of the two equations (28) obtained by elimination of the explicit dependence on $\tan\beta$; we determine the associated value of $\tan\beta$ and run down again the scale evolution equations with these up-dated values of $\mu(0)$ and $\tan\beta$. While this procedure leads to accurate solutions for the second equation (28), thanks to its strong sensitivity to $\mu(0)$, it fails in general to accurately solve the first equation. The inaccuracies originate

from the disparity between the two scales in the problem, m_g and m_Z, and worsen with increasing m_g. For $m_g \leq 200$ GeV, they represent from a few 10 % in favorable cases to 50 %. However, the inaccuracies dramatically increase with larger values of m_g.

Results are shown in figure 1 for case I-a and figure 2 for case III-a. We see (figures a-b) that solutions with $\tan\beta$ of order unity and concomitantly $\mu(0)$ large in comparison to m_g, are generally selected. The variation with θ of these solutions to the electroweak breaking equations depend on the boundary conditions used for Higgs bosons masses. For widely unequal Higgs bosons masses at M_X (cases I or II), with increasing θ, $\tan\beta$ decreases and μ increases; the opposite takes place for equal masses at M_X (case III). The physical (scale m_Z) masses of gauginos, scalars and fermionic superpartners (figures (c), (d), (f)) are all increasing functions of $|\sin\theta|$. For gaugino masses, this is due to the fact that the dilaton term dominates over the moduli term. While the latter term increases with $< T >$ and with δ_{GS}, in all cases that we deal with here, it represents a small, few % fraction of the former and is of negative sign. Note that the moduli contribution is what prevents mass ratios of the three gaugino masses from being θ-independent.

The scalars mass matrices in the interaction basis are diagonal ones, due to our restriction to canonical kinetic terms and the simplified version used for the scale evolution. The values of the averaged traces of the scalars mass matrices are seen to lie at a factor $2-4$ (1) times the basic energy scale, m_g for squarks (sleptons). The third generation masses only are affected by the Yukawa couplings. These contributions shift down the $(3,3)$ scalars masses in comparison to the $(1,1),(2,2)$ masses by a few % for squarks and $10-20\%$ for sleptons. (Since we do not exactly diagonalize the full mass matrices, by taking into account left-right chirality mixing, the foregoing statements are approximate ones.) The top-quark and Higgs boson masses, which have not been plotted in the figures, are also smooth increasing functions of $|\sin\theta|$. As θ sweeps the interval $[0,\pi]$, the top-quark mass in case III, for instance, increases monotonically in the interval $[111-150]$ GeV; the corresponding interval for the tree level Higgs boson mass is $[12-90]$ GeV. For case I, the variation with θ is of opposite sense. The third generation trilinear parameters $A^x(t_Z)$ (figure (e)) are strongly influenced by radiative corrections. For case I, as θ sweeps the interval $[0,\pi]$, $A_t(0)$ regularly decreases from 1.4 to -3.4, while $A_t(m_Z)$ (figure 1 (e)) first levels off then slowly decreases from 4 to 0.6. The parameters $A_{b,\tau}$ are less affected by radiative corrections. This is also the case for the $B-$parameter, for which $B(0)$ regularly decreases from -0.4 to -3.6 while $B(t_Z)$ decreases from -0.3 to -2.5.

The rapid decrease with decreasing θ of the average scalars masses near small θ or $\pi-\theta$ is explained by the fact that the masses of scalars with finite modular weights $|n_\alpha| \geq 1$, may then take negative values at M_X. When these values are large enough in absolute value, as for the case of large $|n_\alpha|$, the repulsive contributions of gaugino interactions, which also get smaller there because of the reduced gauginos masses, are unable to flip the physical scalars masses sign to positive. This results in drastically reduced averaged traces. Such values of θ should be discarded since the associated vacuum would then lead to tachyonic particles. The main agent to avoid such negative squared masses are gaugino loops. These are more effective for squarks than for leptons. The contributions of Yukawa couplings oppose those of gauginos and are obviously larger for u-squarks than d-squarks and sleptons. As a result, the upper bounds on $\cos\theta$ from the constraint of the absence of tachyonic particles are mainly determined by the sleptons masses. These bounds depend somewhat on the modular weights assignments. For $\frac{\theta}{\pi}$ or $|\frac{\theta}{\pi}-1| > 0.2$ no tachyonic particles are present in nearly all three cases described above. This is the reason why we have deleted these intervals in plotting the figures. Finally, we observe that the chargino (neutralino) minimal mass eigenvalues (figure (f)) lie at 3 $(\frac{1}{2})$ times m_g with a $\theta-$dependence similar to that of the wino mass.

The parameter m_g essentially fixes the overall scale for most dimensionful quantities in spite of the fact that the ratio $\frac{m_g}{m_Z}$ is also relevant to the electroweak breaking condition. Increasing m_g in the interval $[100 - 1200]$ GeV, affects slightly $\tan\beta$ and negligibly the dimensionless parameters, A^x, B, while all superpartner mass parameters essentially scale linearly with m_g. This is clearly demonstrated by figure 3.

Let us briefly describe the effect of changing the interval of variation of the angle θ from the first and second quadrants, corresponding to the choice made above, to the third and fourth quadrants. This change leaves scalar masses unchanged but modifies the interference of moduli and dilaton contributions in the other parameters, flipping the signs of A_x, B and leading to gaugino masses of negative sign. Naturally we need to flip the signs of gaugino masses to positive by performing a phase redefinition of the Majorana spinor fields of gauginos. The results for the interval of angles $\theta \geq \pi$ are not qualitatively different from those above in the interval $\theta \leq \pi$.

3.5 Flavor changing parameters

The flavor changing parameters are dimensionless and hence essentially independent of m_g. Their variation with Goldstino angle is depicted in figure 1 for case I-a and figure 2 for case III-a (figures (g)-(l)). Let us observe first that because absolute values are plotted here, the sudden changes of slopes featured by certain curves are simply due to changes of sign in the corresponding flavor changing parameters. The frequent occurrence of such sign flips, as a function of θ, reflects the presence of large cancellations between the boundary conditions and the radiative corrections contributions which vary in opposite sense with variable θ.

Considering first the unmixed chirality parameters, we see that they are decreasing functions of $|\sin\theta|$; sleptons have the fastest slopes and d-squarks come next. The fast decrease near small θ or $\pi - \theta$ is a combined effect of the decreasing averaged scalars masses there and the increasing generation dependent boundary conditions (cf. eqs.(36), (40)). Due to the specific structure of the fermions transformation matrices, the parameters $(\delta_{MM})_{ij}$ are roughly proportional to the differences of modular weights $n_i - n_j$. Turning next to the mixed chirality parameters, we see that they vary rather slowly with θ. This is explained by the fact that the non-universal contributions vary in an opposite sense from the gaugino contributions, as previously discussed in subsection 3.2. Sleptons have the largest slopes because of the smaller gauginos contributions there.

A comparison with experimental bounds in Table 1 reveals that our predicted values in figures 1 and 2 lie safely below the experimental ones, with the exception of δ^x_{12} and $(\delta^x_{LR})_{12}$ for $x = d, e$. We can infer from this comparison lower bounds on θ and $\pi - \theta$. For case I-a, the lower bounds in the d-squarks case lie approximately at 0.2, while the entire interval of θ angles is excluded in the sleptons case. For case III-a, the squarks case entails a larger lower bound of 0.4, while the sleptons case again excludes all angles. The excluded intervals in θ increase in proportion to the size of perturbations away from universality of modular weights and also to the size of modular weights themselves. For the mixed chirality parameters, the model predictions largely exceed experimental bounds, except for $(\delta^e_{LR})_{12}$ for which predictions lie an order of magnitude above the bounds for the entire θ interval.

For a refined comparison one needs to rescale experimental bounds in Table 1 by factors $(\tilde{m}/1000GeV)$ for squarks and $(\tilde{m}/70GeV)^p$ for sleptons, with $p = 2$ for δ^e_{MM} and $p = 1$ otherwise, using the predicted average scalar masses given by figures (d). This correction induces on the squarks bounds a reduction by factors of average size 2 and for sleptons an enhancement by factors of average size 10 (3) for unmixed (mixed) chirality parameters. This will reduce the predictions for squarks, hence will increase the

lower bounds on θ and $\pi - \theta$, but will partially fill in the gap between predictions and experimental bounds for sleptons. The rescaling with respect to the variable x is not very important for squarks, for which the model predictions are compatible with $x \approx 1$. For sleptons, however, a wide gap separates the values occurring in predictions, $x_\gamma \approx 1$, from that used in Table 1, $x_\gamma \approx 0.02$. Since a rescaling of the experimental bounds in Table 1 to $x_\gamma = 1$ is expected to raise these bounds by a factor of order unity [34], this would not modify the above conclusions. We have not pursued here a more quantitative study of the x_γ dependence.

It is interesting to determine the maximal perturbations away from universality of modular weights allowed by the condition of absence of tachyonic particles and the bounds on flavor changing parameters in the first and second generations. We find that perturbing the solutions in ref.[8] for modular weights by a generation non-universality exceeding two units results in flavor changing parameters larger than experimental over the entire interval of θ. This would essentially rule out the model. Detailed results for various cases are presented in Table 2. The three versions of case I are characterized by degenerate first and second generations. The one unit differences of modular weights with respect to the third generation results in contributions of size 10^{-3}, hence roughly the same order as the radiative corrections (compare I-a and I-a'). The comparison of results between cases I-a and I-a' and II-b and II-b' shows us that the radiative corrections from first and second generations may contribute at about the same level as the boundary conditions. Certain changes of signs reflect the presence of strong cancellations between radiative corrections and boundary conditions contributions. The results for case II show that mixed chirality are proportional to difference of modular weights , since going from II-a to II-b enhances the parameters by factor $4-10$. The right chirality parameters are more strongly enhanced because of stronger cancellations effects present there. Finally, we note that the mixed chirality parameters are remarkably stable within factors $2 - 3$.

In order to reduce the flavor changing parameters two main possibilities can be envisaged. One can eventually assign untwisted string modes, with $n_\alpha = -1$, to all three generations or to a subset. This choice might conflict with the requirement of obtaining large scale hierarchies in the fermionic mass matrices. An alternative possibility is to allow for a non-vanishing goldstino matter component. This could significantly reduce the non-universality, since, as seen from eq.(10) the non universal contributions involve a $\cos^2\theta_A$ factor reduction which is stronger than the opposing gauginos loops universality dilution involving $\cos\theta_A$. In this case one would reduce both squarks and sleptons parameters.

4. Conclusions and outlook

The main issue adressed in this paper concerned the flavor changing neutral current constraints imposed on standard-like orbifold models using the duality symmetry approach initiated in refs.[8,19]. We have obtained detailed numerical predictions for the low energy spectrum and flavor changing parameters, essentially covering the entire parameter space of the model. Our results complement those of ref.[19] and establish that all experimental bounds for masses of superpartners scalars and fermions can be obeyed provided the gravitino mass is chosen to lie above roughly 200 GeV. While the flavor changing parameters are controlled at unification scale by a strong dependence on modular weights and goldstino angle the residual contributions left out after radiative corrections are taken into account are proportional to modular weights (differences) and have a smoothed dependence on θ. The relevant flavor changing parameters in the comparison with existing experimental bounds are those associated with the first and second generations, especially sleptons parameters. The experimental bounds can be comfortably respected if θ or $\pi - \theta$ lie above 0.3 and if non-universal perturbations are chosen for modular weights not exceeding two units away from values in solutions compatible with anomalies cancellation and gauge coupling constants unification. Lower bounds of same size are also required in order to avoid tachyonic particles. The main features of the model which imply these properties are the

large gauginos and scalars masses in comparison to the gravitino mass. The framework in which the present work has been developed involves three main shortcomings which we briefly review here in order to motivate a future improved treatment of the problem.

(i) Off-diagonal couplings in the matter Kähler potential. Terms of form, $\delta K = f_{\alpha\bar{\beta}}(M,\bar{M})A_\alpha A_\beta^\dagger + c.c.$, with α, β representing identical gauge group low energy modes but in different generations, are not excluded in general, although these are generically absent in orbifold models. Even when present, such terms can always be transformed away by a linear transformation of superfields, which however will affect the Yukawa couplings. Off-diagonal matter Kähler potentials can also arise if massless string modes mix, so that low energy fields are identified with linear combinations of string modes. Such a situation is commonly encountered when one exploits flat directions to assign VEVs to certain scalars designed to reduce the rank of the gauge group or to truncate the massless spectrum.

(ii) Relative alignment of scalars mass and Yukawa coupling matrices in generation space. On side of gaugino loops radiative corrections, this is the next important item needed to explain naturally small flavor changing parameters. Although the structure of the fermions and scalars superpartners matrices are sensitive to different characteristics of orbifolds (modular weights versus fixed points) it would be desirable to demonstrate on specific examples whether some relationship between them exists and to demonstrate its properties. While the threat from a minimal non-universality which is restricted to diagonal elements of mass matrices has been partially discarded through our present study, one still needs to worry about off-diagonal terms. Such contributions are difficult to avoid when non-renormalizable interactions are taken into account. It is then necessary to verify whether the occurrence of large hierarchies in scalars mass matrices and in Yukawa coupling matrices are compatible and if this can be achieved in orbifold models without postulating an horizontal symmetry.

(iii) CP-violation bounds. Well-known mechanisms contributing to imaginary parts of mass shifts in the neutral mesons systems or to electric dipole moments provide estimates for the imaginary parts of flavor changing parameters which are an order of magnitude smaller than the experimental bounds for real parts. Approximate calculations of the relevant CP-violating phases, $\phi_A = Im(AM_a)$, $\phi_B = Im(B\mu)$, indicate the need of assigning small values to the complex phases in the scalar or auxiliary components of superfields. If such small contributions are confirmed by a more quantitative study, this would then require that dilaton and moduli fields VEVs should stabilize very close to real values, a property which seems to be realized in a natural way in descriptions of supersymmetry breaking incorporating duality symmetries.

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Tables Captions

Table 1. Experimental upper bounds for the δ^x_{MN} matrices of up-squarks (1st line), d-squarks (2d-4th lines) and sleptons (5th line). Our sources are the works of Gabbiani and Masiero (GM) [20], Nir and Seiberg (NS) [27] and Hagelin et al., (HKT) [34]. We set the average squarks mass to $\tilde{m} = 1000$ GeV and sleptons mass to $\tilde{m} = 70$ GeV. The parameters $x_3 = (\frac{M_3}{\tilde{m}})^2$ (squarks) or $x_\gamma = (\frac{M_{\tilde{\gamma}}}{\tilde{m}})^2$ (sleptons) are set to 1 in the bounds of NS [27] and HKT [34] and to $x_3 = 0.49, x_\gamma \approx 0.029$ in those of GM [20]. (Contributions arising from A and μ terms in ref.[20] are neglected.) The notation δ_{ij} is reserved to bounds obtained from box diagrams contributions proportional to the products $\delta_{LL}\delta_{RR}$. Likewise, bounds on δ_{LR} are obtained from contributions involving the products $[(\delta_{LR})_{ij}(\delta_{LR})_{ji}]^{\frac{1}{2}}$. For neutral $K\bar{K}, B\bar{B}, D\bar{D}$ systems, the bounds on $(\delta_{MN})_{ij}$ scale as $\tilde{m}$ at fixed x. For decays $q_i \to q_j + \gamma, l_i \to l_j + \gamma$, the bounds for $(\delta_{MM})_{ij}$ scale as $\tilde{m}^2$ and those for $(\delta_{LR})_{ij}$ as $\tilde{m}$, at fixed x. Thus, except for the sleptons bounds δ_{MM} here, which scale as $\tilde{m}^2$, all other bounds quoted in the table scale as $\tilde{m}$. Note that the new Cleo measurements for $b \to s + \gamma$ yield the bound $(\delta^d_{LR})_{23} = 0.028$.

Table 2. Flavor changing parameters for d-squarks (top half) and sleptons (bottom half) for fixed $m_g = 200GeV, \theta/\pi = 0.35$. The prime indicates that we have omitted radiative corrections in first and second generations which are described in the leading logarithm approximation of eq.(23). The various cases described in Section 3.3 are identified in the first column. The modular weights in the order $\alpha = [Q; U^c; D^c; L; E^c]$ are: case I-a: $n_\alpha = -[221; 332; 221; 223; 223]$; case II-a: $n_\alpha = -[111; 133; 111; 333; 133]$; caseII-b: $n_\alpha = -[023; 133; 023; 023; 133]$; case III-a:$n_\alpha = -[123]$; case III-b:$n_\alpha = -[112]$;

Figures Captions

Figure 1. Plot as a function of θ/π for $m_g = 200$ GeV of the model parameters in Case I-a: (a) $\tan\beta$; (b) $\mu(0)$; (c) standard model gauginos masses $M_a(t_Z)$ in the order $a = 3, 2, 1$ from top to bottom ; (d) Average trace of left times right chirality mass submatrices, $\tilde{m}^x = [Trace(\tilde{M}^2_{LL}(t_Z))Trace(\tilde{M}^2_{RR}(t_Z))]^{\frac{1}{4}}$, for d-squarks (continuous), u-squarks (dot-dashed) and sleptons (dashed); (e) supersymmetry breaking parameters $A_x(t_Z)$ for x=t (continuous), b (dot-dashed), τ(dashed), and $B(t_Z)$ (dotted); (f) minimum masses of charginos (continuous) and neutralinos (dot-dashed); (g)-(i) $\delta^x_{ij} = |(\delta^x_{LL})_{ij}(\delta^x_{RR})_{ij})|^{\frac{1}{2}}, \quad [x = d, e, u]$ for $(i, j) = (1, 2)$ (figure (g)), $(1, 3)$(figure (h)) and $(2, 3)$ (figure (i)); (j)-(l) $\delta'^x_{ij} = |(\delta^x_{LR})_{ij}(\delta^x_{LR})_{ji})|^{\frac{1}{2}}, \quad [x = d, e, u]$ for $(i, j) = (1, 2)$ (figure (j)), $(1, 3)$(figure (k)) and $(2, 3)$ (figure (l)). Curves in each of figures (g)-(l) refer to d-squarks (continuous), sleptons (dot-dashed) and u-squarks (dashed). Note that in figures (g)-(l) the discontinuities in slopes occurring for certain curves signal the occurrence in these semi-logarithmic plots of changes of signs for quantities in ordinate.

Figure 2. Same conventions as figure 1 in Case III-a.

Figure 3. Plot versus m_g for fixed $\theta = 0.35\pi$ of low energy parameters in Case I-a, using the same conventions for curves as in figure 1.

TABLE 1

Sector	$(\delta_{MM})_{12}$	$(\delta_{MM})_{13}$	$(\delta_{MM})_{23}$	δ_{12}	δ_{13}	δ_{23}	$(\delta_{LR})_{12}$	$(\delta_{LR})_{13}$	$(\delta_{LR})_{23}$
U (NS) ref.[27]	0.1			0.04			0.06		
D (NS) ref.[27]	0.05	0.1		0.006	0.04		0.008	0.06	0.04
D (GM) ref.[20]	0.035	0.23		0.0051	0.074		0.007	0.1	0.12
D (HKT) ref.[34]	0.1	0.27	47	0.006	0.073		0.0044	0.14	0.071
L (GM) ref.[20]	$8.96 10^{-4}$	7.67	7.08				$1.55 10^{-6}$	$2.24 10^{-1}$	$2.06 10^{-2}$

TABLE 2

Case	$(\delta_{LL})_{12}$	$(\delta_{LL})_{13}$	$(\delta_{LL})_{23}$	$(\delta_{RR})_{12}$	$(\delta_{RR})_{13}$	$(\delta_{RR})_{23}$	$(\delta_{LR})_{12}$	$(\delta_{LR})_{13}$	$(\delta_{LR})_{23}$
D-SQUARKS									
Ia	3.210^{-3}	-3.610^{-4}	3.010^{-3}	3.710^{-4}	4.710^{-4}	-6.610^{-4}	9.210^{-5}	9.410^{-5}	4.210^{-4}
Ia'	-4.010^{-4}	-5.110^{-4}	3.710^{-3}	-4.310^{-4}	4.310^{-4}	-5.110^{-4}	9.210^{-5}	9.410^{-5}	4.210^{-4}
Ib	3.210^{-3}	-8.510^{-4}	5.210^{-3}	3.310^{-5}	4.110^{-4}	-4.210^{-4}	7.910^{-5}	8.410^{-5}	3.710^{-4}
IIa	2.910^{-3}	-6.410^{-4}	4.310^{-3}	6.710^{-5}	3.110^{-4}	6.810^{-5}	9.810^{-5}	1.110^{-4}	4.710^{-4}
IIb	1.110^{-2}	-9.710^{-4}	-5.810^{-3}	8.410^{-3}	1.310^{-4}	8.810^{-4}	1.210^{-4}	1.210^{-4}	5.210^{-4}
IIb'	7.310^{-3}	-1.110^{-3}	6.610^{-3}	7.810^{-3}	1.110^{-4}	9.710^{-4}	1.210^{-4}	1.210^{-4}	5.210^{-4}
IIIa	5.810^{-3}	-1.310^{-3}	7.210^{-3}	3.010^{-3}	1.610^{-4}	7.210^{-4}	9.210^{-5}	1.010^{-4}	4.510^{-4}
IIIb	1.910^{-3}	-1.210^{-3}	6.810^{-3}	-2.310^{-4}	1.610^{-4}	7.210^{-4}	8.510^{-5}	1.010^{-4}	4.610^{-4}
SLEPTONS									
Ia	7.110^{-3}	2.810^{-4}	-7.710^{-4}	-1.710^{-4}	-4.810^{-4}	1.110^{-2}	3.410^{-5}	4.110^{-5}	6.010^{-4}
Ia'	2.310^{-4}	-1.510^{-4}	5.810^{-3}	-2.210^{-4}	-4.810^{-4}	1.110^{-2}	3.410^{-5}	4.110^{-5}	6.010^{-4}
Ib	1.010^{-2}	4.110^{-4}	-2.810^{-3}	-1.810^{-4}	-6.910^{-4}	1.410^{-2}	3.610^{-5}	4.310^{-5}	6.310^{-4}
IIa	7.010^{-3}	6.810^{-4}	-6.110^{-3}	2.610^{-2}	3.110^{-4}	3.710^{-5}	4.710^{-5}	3.210^{-5}	4.510^{-4}
IIb	2.210^{-2}	3.410^{-4}	-1.810^{-4}	2.610^{-2}	3.110^{-4}	6.110^{-5}	5.510^{-5}	3.310^{-5}	4.910^{-4}
IIb'	1.410^{-2}	-1.310^{-4}	6.010^{-3}	2.610^{-2}	3.010^{-4}	1.110^{-4}	5.510^{-5}	3.310^{-5}	4.910^{-4}
IIIa	1.810^{-2}	3.510^{-4}	4.710^{-4}	1.810^{-2}	-7.510^{-4}	1.510^{-2}	4.810^{-5}	4.010^{-5}	6.010^{-4}
IIIb	6.910^{-3}	2.110^{-4}	2.210^{-3}	-2.010^{-4}	-6.410^{-4}	1.310^{-2}	3.110^{-5}	3.510^{-5}	5.410^{-4}